OPTIMAL LINEAR ESTIMATION AND SUBOPTIMAL NUMERICAL SOLUTIONS OF DYNAMICAL SYSTEMS CONTROL PROBLEMS

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ABSTRACT

The proposed procedure applies to the generation of open loop suboptimal numerical solutions in dynamical systems optimal control problems. The approximation of the control by a function dependent upon a finite number of parameters, and the use of a linear perturbation scheme associated with a direct search criterion reduce the problem to one of parameter optimization, in each iteration. A stochastic approach to establish the search criterion and to treat the errors due to the first order approximation makes it possible to arrive at the search increment using optimal linear estimation.

Keywords: Suboptimal Control, Numerical Methods in Optimal Control, Trajectory and Transfer Orbits Optimization, Linear Optimal Estimation.

1. INTRODUCTION

The optimization of open loop solutions in dynamical systems optimal control problems is fundamental during the design phase for the definition of a nominal solution, which is not only acceptable in terms of problem constraints, but is also the best in terms of an index of performance. For most of the problems of practical interest a numerical treatment is necessary, including those of controlling trajectories and attitude maneuvers of space vehicles.

This work presents a numerical procedure for the treatment of optimal control problems, combining three approaches: (i) an approximation of the control by a function dependent upon a finite number of parameters, leading to a suboptimal control problem; (ii) a linear perturbation scheme associated with a direct search criterion, reducing the problem to one of parameter optimization in each iteration; and (iii) a stochastic interpretation of the search increment and of the errors due to the linear approximation, employing optimal linear estimation to arrive at the search increment.

The choice of a first order direct search method (Ref. 1) is dictated by the characteristic of good numerical behavior, even when the initial guesses are not close to the solution. The choice of the control suboptimal approximation is necessary to reduce to a parameter optimization the numerical treatment of the dynamical problem. Besides that, this choice meets the objective of saving processing time and computer memory space (Refs. 3-6).

The use of optimal linear estimation to find the search increment has the objective of testing the validity of an alternative numerical tool to solve the parameter optimization problem associated to each iteration. The procedure presented in this work is the result of the extension and refinement of results previously presented by one of the authors (Ref. 2).

To evaluate numerical performance, the example chosen was one that has been used by many authors (Refs. 3-6) in testing optimal and suboptimal procedures that apply to dynamical systems control problems. It consists of a simplified minimum time Earth to Mars orbit transfer with low thrust of fixed magnitude and controlled direction.

2. PROBLEM STATEMENT

The proposed numerical procedure applies to the solution of optimal control problems with performance index, dynamical and boundary constraints given by:

$$IP = G(x_0, x_f, t_0, t_f) \tag{1}$$

$$\dot{x} = f(x,u,t) \tag{2}$$

$$C(x_0, x_f, t_0, t_f) = 0 \tag{3}$$

where x is the n-component state vector; u is the m-component control vector; t stands for time; C(.) is the N_C ($N_C < 2n+2$) column vector constraint function of the initial and final values of time (t_0 and t_f) and of the state (x_0, x_f).

Thus, the objective is the optimization of the index of Eq. 1 (either minimization or maximization), under the restriction that Eqs. 2,3 will be satisfied on the solution. Taking the approach of the so called direct search methods, a numerical procedure is proposed in the following sections. It makes use of a suboptimal approximation for the control, and of an optimal linear estimation for the determination of the search increment.

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3. SUBOPTIMAL SCHEME

If $\overline{x}_0$, $\overline{x}_f$, $\overline{t}_0$, $\overline{t}_f$ are starting guesses or values previously obtained, the solution of the problem of Eqs. 1-3 can be seen in a typical iteration, under a direct search approach (Refs. 1,2), as the optimization of

$$IP = G(\overline{x}_0+dx_0,\ \overline{x}_f+dx_f,\ \overline{t}_0+dt_0,\ \overline{t}_f+dt_f) \tag{4}$$

subject to

$$(\dot{\overline{x}} + \delta\dot{x}) = f(\overline{x} + \delta x,\ \overline{u} + \delta u,\ t) \tag{5}$$

$$C(\overline{x}_0+dx_0,\overline{x}_f+dx_f,\overline{t}_0+dt_0,\overline{t}_f+dt_f) = \alpha\, C(\overline{x}_0,\overline{x}_f,\overline{t}_0,\overline{t}_f) \tag{6}$$

where $0 \leq \alpha < 1$; and since a linear perturbation scheme is to be adopted, the increments have to be sufficiently small, corresponding to first order variations, such that

$$dx = \dot{\overline{x}}\ dt + \delta x \tag{7}$$

$$\delta\dot{x} = f_{\overline{x}}(\overline{x},\overline{u},t)\cdot\delta x + f_{\overline{u}}(\overline{x},\overline{u},t)\cdot\delta u \tag{8}$$

$$\dot{S}(t,\overline{t}_0) = f_{\overline{x}}(\overline{x},\overline{u},t)\cdot S(t,\overline{t}_0)\ ,\quad S(\overline{t}_0,\overline{t}_0) = I \tag{9}$$

where $f_{\overline{x}}(\overline{x},\overline{u},t)$ and $f_{\overline{u}}(\overline{x},\overline{u},t)$ are the matrices of first order partial derivatives with respect to state and control, evaluated on the over bar values; and $S(t,\overline{t}_0)$ is the associated transition matrix, which gives

$$\delta x(t) = S(t,\overline{t}_0)\cdot\delta x_0 + \int_{\overline{t}_0}^{t} S(t,s)\cdot f_{\overline{u}}(\overline{x},\overline{u},s)\cdot\delta u(s)\cdot ds \tag{10}$$

relating first order variations of the state at any time t with first order variations of the initial state and of the control time history.

With the objective of transforming the determination of the search increments, in each iteration, into a problem of parameter optimization, a suboptimal approximation is taken for the control (Refs. 3,4). It consists in replacing the control by a function $U(p,t)$ dependent upon a finite number of parameters. This suboptimal control is modeled by arcs, with no restriction of continuity at the junction points, leading to arcs of the state trajectory possibly connected through corners. The number of control arcs (K) and the number of parameters defining each control arc (j_k+1) are a matter of previous choice. Following Ref. 5, in the interval correspondent to the k^{th} control arc, it results:

$$\delta u(t) = \sum_{j=0}^{j_k}\left(\frac{\partial}{\partial p_j^k}U(\overline{p}_0^k,\overline{p}_1^k,\ldots,\overline{p}_{j_k}^k;t)\right)\cdot dp_j^k = \sum_{j=0}^{j_k} D_j^k(\overline{p}^k;t)\cdot dp_j^k \tag{11}$$

where

$$u_i(t) = U_i^k(p_{0i}^k,p_{1i}^k,\ldots,p_{j_k i}^k;t)$$

where $i = 1,2,\ldots,m$; and, for $\overline{t}_k < t < \overline{t}_{k+1}$, p_j^k are mx1 vectors of parameters with $j = 0,1,\ldots,j_k$, $k = 0,1,\ldots,K-1$. Thus, from Eqs. 8,9 it results:

$$\delta x(\overline{t}_{k+1}^-) = S(\overline{t}_{k+1}^-,\overline{t}_k^+)\cdot\delta x(\overline{t}_k^+) + \sum_{j=0}^{j_k}\left(\int_{\overline{t}_k^+}^{\overline{t}_{k+1}^-} S(t,s)\cdot f_{\overline{u}}(\overline{x},\overline{U}^k,s)\cdot D_j^k(\overline{p}^k,s)\cdot ds\right)\cdot dp_j^k \tag{12}$$

where the upper minus and plus signs indicate the values just to the left and to the right, respectively.

However, at the junction point $\overline{t}_k$, where a corner may exist, it is necessary to impose (Ref. 1):

$$\dot{\overline{x}}(\overline{t}_k^-)\cdot dt_k+\delta x(\overline{t}_k^-) = \dot{\overline{x}}(\overline{t}_k^+)\cdot dt_k+\delta x(\overline{t}_k^+) \tag{13}$$

Taking the results given by Eqs. 12,13 back to the problem of Eqs. 4,6, it results the following associated problem in a typical iteration, after some algebraic manipulations (Ref. 5):

$$\text{Optimize: } L(v) \tag{14}$$

$$\text{Subject to: } M(v) = \alpha\cdot\overline{M} \tag{15}$$

where the over bar variables have been omitted, since they are constant in each iteration; $L(v)$ and $M(v)$ replace $G(\overline{x}_0+dx_0,\overline{x}_f+dx_f,\overline{t}_0+dt_0,\overline{t}_f+dt_f)$ and $C(\overline{x}_0+dx_0,\overline{x}_f+dx_f,\overline{t}_0+dt_0,\overline{t}_f+dt_f)$, respectively; and

$$v^T \triangleq \left[(dp^0)^T:(dp^1)^T:\ldots:(dp^{K-1})^T:\delta x_0^T:(dt_0,dt_1,\ldots,dt_f)\right]$$

From Eqs. 14,15 it is clear that the problem has been reduced to the optimization of the parameters correspondent to the search increments, in each iteration. This has to be done satisfying the linear perturbation hypothesis and the criterion of getting closer to the suboptimal solution, in the next iteration.

4. PROPOSED PROCEDURE

In the direct search procedure to be proposed, the increment vector in the problem of Eqs. 14,15 is taken as the sum of two other increments,

$$v = v^1 + v^2 \tag{16}$$

which are to be found using optimal linear estimation and meeting the requirements of the search criterion. These increments translate the objective of getting closer to meet the constraints (v^1) and the suboptimal value of the index of performance (v^2).

4.1 Determination of First Increment

To find v^1, a first order series expansion of the left hand side of Eq. 15, about the values of the previous iteration $(\overline{v} = 0)$, is taken, resulting

$$M_{\overline{v}}\cdot v^1 + o(2) = (\alpha-1)\cdot\overline{M} = -q\overline{M} \tag{17}$$

where $M_{\overline{v}}$ is the matrix of first order partial derivatives, and o(2) represents the high order terms in the expansion. If e_{a_i} is the maximum admissible error in the satisfaction of the i^{th} of Eqs. 17, it is reasonable to model o(2) by a Gaussian random noise vector E_M^1 of uncorrelated components, given by:

$$E\left[E_{M_i}^1\right] = 0,\ E\left[(E_{M_i}^1)^2\right] = 1/9 \cdot e_{a_i}^2,\ i=1,2,\ldots,N_C \tag{18}$$

where $E[.]$ means expectation. Hence a condition for determining v^1 is now given by

$$\frac{\partial}{\partial v} M_i(\overline{v}) \cdot v^1 + E_{M_i}^1 = -q_i\ \overline{M}_i \tag{19}$$

Considering that the right hand side of Eq. 19 has the magnitude of a first order term, and to be consistent with the hypothesis of linear perturbation, it is also reasonable to adopt the value of q_i as given the following empirical criterion:

$$q_i = \mathrm{Min} \cdot \{q_{ij} : q_{i0} = 1,\ q_{i1}^2\ \overline{M}_i^2 = (\beta e_{a_i})^2\} \tag{20}$$

where $i=1,2,\ldots,N_C$; and $\beta \gg 1$ is an adjustable convergence parameter consistent with increments within the linear perturbation region limits. To complete the conditions for an estimate of v^2, the a priori piece of information is considered to be:

$$\overline{v}^1 = 0 = v^1 + \eta \tag{21}$$

$$E\left[\eta_j\right] = 0,\ E\left[\eta_j \eta_k\right] = (\overline{\sigma}_j^1)^2 \cdot \delta_{jk} = \overline{P}_{jk}$$

where $j,k=1,2,\ldots,N_p$, the number of parameters; and δ_{jk} is the Kronecker symbol. To evaluate the $\overline{\sigma}_j^1$, statistical consistency will be imposed by maximizing the probability of occurence of the observation residues given by Eq. 19 (Ref. 7), resulting

$$\sum_{j=1}^{N_p} \left(\frac{\partial}{\partial v_j} M_i(\overline{v}) \cdot \overline{\sigma}_j^1\right)^2 + 1/9 \cdot e_{a_i}^2 = q_i^2\ \overline{M}_i^2 \tag{22}$$

where $i=1,2,\ldots,N_C$. Adopting for each $\overline{\sigma}_j^1$ a criterion of equal opportunity to contribute to the satisfaction of the consistency requirement, it results:

$$\left(\frac{\partial M_i}{\partial v_j}(\overline{v}) \cdot \overline{\sigma}_j^1\right)^2 = (q_i^2\ \overline{M}_i^2 - 1/9 \cdot e_{a_i}^2)/N_p \tag{23}$$

and applying, for each $j=1,2,\ldots,N_p$, a least squares fitting:

$$(\overline{\sigma}_j^1)^2 = \left(\sum_{i=1}^{N_C} \left(\frac{\partial M_i}{\partial v_j}(\overline{v})\right)^2 \cdot (q_i^2\ \overline{M}_i^2 - 1/9 \cdot e_{a_i}^2)\right) \Big/ \left(N_p \cdot \sum_{i=1}^{N_C} \left(\frac{\partial M_i}{\partial v_j}(\overline{v})\right)^4\right) \tag{24}$$

as far as the value given by Eq. 24 is positive, and $(\overline{\sigma}_j^1)^2 = 0$ if the value given by this equation is negative.

Finally, applying a Kalman filtering or, equivalently, a least squares with a priori piece of information, to Eqs. 19,21, an estimate $\hat{v}^1$ of v^1 is obtained.

$$\hat{v}^1 = K \cdot (Q \cdot \overline{M}) \tag{25}$$

$$K = P \cdot M_{\overline{v}}^T \cdot (R^1)^{-1} \tag{26}$$

$$P = \overline{P} - \overline{P} \cdot M_{\overline{v}}^T \cdot (M_{\overline{v}}\ \overline{P} M_{\overline{v}}^T + R^1)^{-1} \cdot M_{\overline{v}} \cdot \overline{P} \tag{27}$$

$$Q \triangleq \mathrm{diag}.\left[-q_i,\ i=1,2,\ldots,N_C\right],\ E\left[\eta\eta^T\right] \triangleq \overline{P},\ E\left[E_M^1 E_M^{1T}\right] \triangleq R^1.$$

Notice that, since the noise E_M^1 is of uncorrelated componentes, the vector of observations (Eq. 19) can be processed component by component, avoiding the need of matrix inversion.

4.2 Determination of Second Increment

To find v^2, the idealized objective of having this increment vector in the gradient direction of the performance index is considered. Thus, if the problem is for example one of minimization, it would be convenient to have:

$$v^2 = -p \cdot L_{\overline{v}}^T,\quad p > 0 \tag{28}$$

However, a compromise has to be taken to assure convergence to the constraints. This can be done if it is imposed that

$$\frac{\partial}{\partial v} M_i(\overline{v}) \cdot (v^1 + v^2) + E_{M_i}^2 = -q_i\ \overline{M}_i \tag{29}$$

where the error $E_{M_i}^2$ is chosen to guarantee the possibility of having $p > 0$, without loosing the convergence on the constraints. Based on these considerations, $E_{M_i}^2$ is taken as a Gaussian random noise of uncorrelated components and statistics given by

$$E\left[E_{M_i}^2\right] = 0,\ E\left[(E_{M_i}^2)^2\right] = (\beta e_{a_i}/3)^2/\gamma^2 \tag{30}$$

where $i=1,2,\ldots,N_C$, and $\gamma > 1$ is a convergence parameter to be adjusted. Now, from the substitution of values of Eqs. 19,28 in Eq. 29, it results

$$M_{\overline{v}} \cdot (-p\ L_{\overline{v}}^T) + (E_M^2 - E_M^1) = 0 \tag{31}$$

which is the desired observation relationship.

However, since the requirement is to get closer to the suboptimum index of performance without compromising the convergence to constraints, it is reasonable to consider the following *conditioned realization* of this relationship:

$$\frac{\partial M_i}{\partial v}(\overline{v}) \cdot (-p_0 \cdot L_{\overline{v}}^T) - E_M^1 = -q_{i1} \cdot \overline{M}_i/\gamma \tag{32}$$

and p_0 is then estimated by a least squares fitting, resulting

$$\hat{p}_0 = (L_{\overline{v}} \cdot M_{\overline{v}}^T \cdot (R^1)^{-1} \cdot M_{\overline{v}} L_{\overline{v}}^T)^{-1} \cdot (L_{\overline{v}} \cdot M_{\overline{v}}^T \cdot (R^1)^{-1} \cdot Q_1 \cdot \overline{M})/\gamma \tag{33}$$

where $Q_1 \triangleq \mathrm{diag}.\left[-q_{i1},\ i=1,2,\ldots,N_C\right]$.

Finally, the following estimate is taken for v^2:

$$\hat{v}^2 = -/\hat{p}_0/\cdot L_{\overline{v}}^T \tag{34}$$

4.3 Checking Conditions

There are three phases of convergence. The first is a coarse phase, where priority is given to the requirement of getting closer to satisfaction of constraints $(/M(\overline{v}+\hat{v})/\lesssim/\overline{M}/)$. In this phase, the value of q_{i1}, as given by Eq. 20, is less than one $(q_{i1}<1)$. If reduction of the constraints is not met, the value of β is decreased before proceeding to a new iteration. Before summing the increments $\hat{v}^1$ and $\hat{v}^2$ to obtain $\hat{v}$, the following verification has to be made:

$$\hat{v}^1+\gamma\hat{v}^2 \neq \eta^1+\gamma\eta^2 \; L_{\overline{v}}^T \tag{35}$$

where $\neq$ means *not of the order of;* η^1 and η^2 are the errors in the estimates of the increments. This verification is done to avoid a needless effort in the noise region. To reduce it to a usable form, the statistics estimated for η^1 and η^2 is used (Eqs. 27,33). This is done considering the 3σ limits:

$$L_{\overline{v}}\cdot(\hat{v}^1+\hat{v}^2) < +3\left(\sum_{i=1}^{N_p} /L_{\overline{v}_i}/\cdot\hat{\sigma}_i^1 + L_{\overline{v}}L_{\overline{v}}^T\cdot\hat{\sigma}^2\right) \tag{36}$$

where $\hat{\sigma}_i^1$ is the standard deviation, as given by the i^{th} diagonal term of the estimated covariance matrix of the errors in the estimate of v^1 (Eq. 27); and $\hat{\sigma}^2$ is the standard deviation of the error in the estimate of p_0 (Eq. 33). If the condition of Eq. 36 is not verified, the search increment is taken equal to $\hat{v}^1 (\hat{v} = \hat{v}^1)$.

The second is a fine phase of convergence and is characterized by the condition of all the q_{i1}, as given by Eq. 20, greater than or equal to one $(q_{i1}\geq 1,\ i=1,2,\ldots,N_C)$ and the $q_i=q_{i0}=1$. In this phase, the following conditions are to be satisfied:

$$(\overline{M}+M_{\overline{v}}\cdot\hat{v})^T\cdot(\overline{M}+M_{\overline{v}}\cdot\hat{v}) \leq \sum_{i=1}^{N_C}(\beta e_{a_i})^2 \tag{37}$$

during all the iterations in the fine phase; and if for all $i=1,2,\ldots,N_C$, it is true that

$$q_{i1}\overline{M}_i/\gamma \geq \overline{M}_i - e_{a_i} \tag{38}$$

then it is necessary that

$$L_{\overline{v}}\cdot(v^1+v^2) = L_{\overline{v}}\cdot(\hat{v}^1+\eta^1+(-/\hat{p}_0/+\eta^2)\cdot L_{\overline{v}}^T) < 0 \tag{39}$$

where, under the 3σ uncertainty given by the estimates, Eq. 39 is to be interpreted as:

$$L_{\overline{v}}\cdot\hat{v} < -3\left(\sum_{i=1}^{N_p}/L_{\overline{v}_i}/\cdot\hat{\sigma}_i^1 + L_{\overline{v}}\cdot L_{\overline{v}}^T\cdot\hat{\sigma}^2\right) \tag{40}$$

In the fine phase, if Eq. 37 — or, when applicable, Eq. 40 — is not met and if the e_{a_i} are all greater than the e_{m_i} $(e_{a_i} > e_{m_i}$, where e_{m_i} is the minimum error in the satisfaction of the i^{th} constraint), the values of the e_{a_i} are reduced, proceeding to an iteration in the third phase of convergence. Whenever in the fine phase the value of the e_{a_i} are all less than or equal to the e_{m_i} $(e_{a_i} < e_{m_i})$, convergence has been reached.

The third phase of convergence is a coarse phase inside the linear perturbation region. In the equations used to calculate the search increment, q_i and q_{i1} are forced to be equal to one $(q_i = q_{i1} = 1)$ in this phase, instead of being taken as given by Eq. 20. However, Eq. 20 is still used to verify when it is necessary to shift back to a fine phase. Since this is only a linear region coarse phase, the checking condition correspondent to Eq. 36 has to be used. For the same reason, priority is given to convergence to the constraints. If this requirement is not met in an interation of the third phase, the values of the q_{i1} in Eq. 33 are reduced only inside that iteration, until constraint convergence is attained.

5. NUMERICAL TESTING

The problem chosen to evaluate the procedure performance is that of a simplified minimum time Earth to Mars orbit transfer, with low thrust of fixed magnitude and controlled direction (Refs. 1,3-6), as given bellow.

$$\text{Minimize: IP} = t_f \tag{38}$$

$$\text{Subject to: } \dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3^2/x_1 - \mu/x_1^2 + T\sin\beta/(m_0-\dot{m}t) \tag{39}$$

$$\dot{x}_3 = -x_2x_3/x_1 + T\cos\beta/(m_0-\dot{m}t)$$

$$x_1(t_0)=1.0,\ x_2(t_0)=0.0,\ x_3(t_0)=1.0$$

$$x_1(t_f)=1.523679,\ x_2(t_f)=0.0,\ x_3(t_f)=0.81012728$$

where x_1 is the radial distance from the Sun to the spacecraft; x_2, the radial velocity; x_3, the tangential velocity; T, the thrust magnitude; m, the mass of the spacecraft $(m_0 = m(t_0))$; μ, the gravitational constant; and β, the control. In normalized units, $\mu = 1.0$, $m_0 = 1.0$, $\dot{m} = 0.074800391$, $T = 0.14012969$.

Table 1 and Figure 1 show the results obtained when the control is approximated by four straight line segments, with no restriction of continuity at the junction points. In this case, in the interval correspondent to the k^{th} segment, the control approximation is given by:

$$U^k(p_0^k,\ p_1^k;\ t) = p_0^k + ((p_1^k-p_0^k)/(t_{k+1}-t_k))\cdot t \tag{41}$$

where $k = 0,1,2,3$; and t_k, $k > 0$, are among the parameters to be optimized $(t_f = t_4)$.

6. CONCLUSIONS

The approach of approximating the control by a function dependent upon a finite number of parameters, in association with a first order direct search method, reduces the problem to one of parameter optimization, in each iteration. This gives to suboptimal procedures using this approach, as the one presented in this work, the

Table 1

NUMBER OF ITERATIONS (NI) AND CONSTRAINT ACCURACY			
NI	$/\Delta\hat{x}_1(t_f)/$	$/\Delta\hat{x}_2(t_f)/$	$/\Delta\hat{x}_3(t_f)/$
14	1.15862E-07	8.13731E-05	4.63165E-05

CONVERGENCE PARAMETERS
β=1.E03, γ=1.85, $e_{a_i}=e_a$=1.E-04, $e_{m_i}=e_m$=1.E-07

CONTROL AND TIME PARAMETERS		
Symbol	Initial Guess	Converged Value
p_0^0	.000000	.521974
p_1^0	1.17810	1.00935
t_1	.850000	.924517
p_0^1	1.17810	1.05665
p_1^1	2.35619	2.26314
t_2	1.70000	1.70891
p_0^2	2.35619	2.57549
p_1^2	3.53429	4.38308
t_3	2.55000	1.88816
p_0^3	3.53429	5.23151
p_1^3	4.71239	5.23151
t_f	3.40000	3.33762

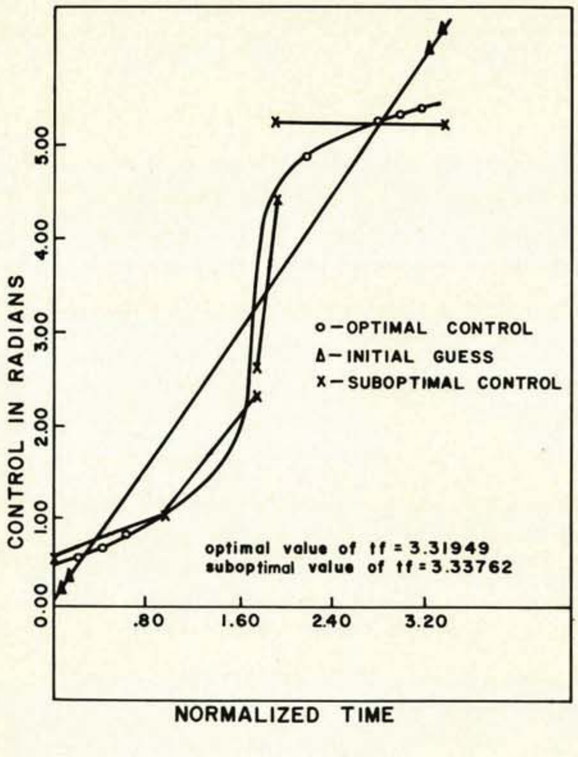

Figure 1. Control versus Time

special features of: (i) saving computer memory space, when compared to first order direct search optimal procedures; and (ii) freedom in the choice of the suboptimal control function form, including those represented by arcs with discontinuities at the junction points.

The use of optimal linear estimation to obtain the search increment is intended to keep the following additional features, exhibited by other suboptimal procedures found in the literature (Refs. 3-6): (i) of saving processing time, when compared to first order direct search optimal procedures; and (ii) of giving suboptimal results which are close in quality to those obtained with optimal procedures. The results of the numerical test give a good indication that the procedure presented attains these additional features.

However, aside from the referred improved features, the use of optimal linear estimation leads to a procedure with the specific characteristic of greatly reducing the number and of simplifying the use of the adjustable parameters needed to control convergence. This happens due to the fact that either the conceptual meaning of these parameters ($\beta,\gamma,e_{a_i},e_{m_i}$) is made clear in the linear estimation problem associated to each iteration, or they are related to statistics noise (η) adaptively determined.

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