APPLICATION OF DYNAMIC PROGRAMMING TO STATION ACQUISITION OF A GEOSTATIONARY SATELLITE

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ABSTRACT

This paper deals with multiple burn apogee maneuver strategies for geostationary satellites. We present an efficient method based on dynamic programming which is suited to the optimization of orbit corrections. The method we present has been designed in order to cope with the curse of dimensionality proper to dynamic programming. The method is split in two parts. First a simplified orbit correction problem which corresponds to impulsive approximation is solved using a backward dynamic programming procedure; In the second part this solution is used to initialize an iterative procedure which looks for an optimum of the low thrust case. The elementary process of the iterative procedure is a backward dynamic programming process. This method is used to solve an highly constrained problem submitted by the C.N.E.S. with limitation on the total time for station acquisition, time interval between apogee maneuvers, visibility to ground stations and location of the spacecraft in the orbit with respect to the sun.

Keywords : Station Acquisition, Geostationary Satellites, Multiple Burn Maneuver, Maneuver Optimization, Dynamic Programming.

1. INTRODUCTION

Three axis stabilized satellites, like television satellite TDF1, have a low thrust propulsion system which can be restartable. As a consequence, various apogee maneuver strategies can be considered for station acquisition. In the particular case of TDF1 the injection from the elliptical transfer orbit to a near-synchronous orbit is accomplished with a 400 N apogee engine. An optimal strategy minimizes the propellant consumption subject to some mission constraints. In this paper we will be concerned with the following constraints : limitation on the total time for station acquisition, time interval between two apogee maneuvers to facilite orbit determination, visibility to ground stations (the longitude of the satellite during the maneuvers must be included with a certain range), location of the spacecraft in the orbit with respect to the sun (this constraint ensures accurate attitude determination and power generation during the maneuvers). In order to optimize apogee maneuvers one has to answer to the following questions:
What is the optimal number of burns?
What are the starting times of burns (and as a corollary what are the numbers of drift orbits between the burns)?
What are the burn durations?
Maneuver optimization is a non-linear optimization problem with equality and inequality constraints. This is a complex problem and it is very difficult to find an optimal burn strategy especially in the case where the mission constraints are very strong. Nevertheless we know that multi-burn apogee maneuver strategies permit to reduce long arc burning losses, to satisfy the solar constraints and to compensate thrust errors (ref. 3). Methods based on local optimization around a solution of the impulsive problem or calculus of variations have been proposed to solve this problem, (ref. 1), (ref. 2). We present now our method.

1.1 Principle of the method

The method we propose consists of two parts. First a backward dynamic programming procedure is used to solve a simplified orbit correction problem corresponding to impulsive approximation. This approximation is still valid for a 400 N engine (ref. 1). The resulting optimal apogee burn strategy gives the number of burns, their respective modules and the number of drift orbits between them. In the second part this solution is used to initialize an iterative procedure which looks for an optimum of the low thrust case. The elementary process of the iterative procedure is a local backward dynamic programming process, i.e. the method iterates via backward dynamic programming around strategies resulting of previous computations until an optimum is found with a given precision. This feature permits to reduce the curse of dimensionality. In this study we consider only the deterministic case, nevertheless the method that we present can be extended to take into account thrust uncertainties. The next section presents the backward dynamic programming process for impulsive maneuver optimization. Section 3 is devoted to the iterative procedure applied to the low thrust case. In section 4 we present some results obtained in the case of an highly constrained problem.

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2. IMPULSIVE MANEUVER OPTIMISATION

In this section we consider a simplified orbit correction problem which corresponds to impulsive approximation. This approximation permits to reduce the dimension of the state vector like the dimension of the decision vector. We note that with impulsive thrust the velocity of the spacecraft varies instantaneously without any change of the satellite position. We suppose also that the transfer orbit belongs to the equatorial plan and that all the burns take place at the apogees and the thrust vectors are parallel to the velocity vectors. So the problem is : how to split the increment of velocity $\Delta V = V_g - V_t$? Where V_g is the velocity of the satellite on the geostationary orbit and V_t is the velocity at the apogee of the transfer orbit.

We give now the system equations which describe the dynamic behavior of the satellite :

$$E(k+1) = C(k) \,.\, (1 - E(k)) \tag{1}$$

$$V(k+1) = V(k) + \Delta V(k) \,.\, E(k) \tag{2}$$

$$L(k+1) = L(k) + H\,(V(k+1)) \tag{3}$$

Where k, which is called the stage variable, is relative to the kth apogee, E(k) is a state variable relative to whether there is or there is not a burn at the kth apogee, i.e. if E(k) = 0 there is no burn, if E(k) = 1 there is a burn. C(k) is an artificial decision variable, $C(k) \in \{0,1\}$. We will give further comments on this variable in the following of this section. V(k) is the satellite velocity relative to the kth apogee, $\Delta V(k)$ is the module of the velocity increment which occurs at the kth apogee, L(k) is the longitude at the kth apogee, H(V(k+1)) is the drift between the kth apogee and the k+1 th apogee, it is a function of the satellite velocity. In the following the state vector of components E(k), V(k) and L(k) will be noted X(k) and the decision vector of components C(k) and $\Delta V(k)$ will be noted U(k). The system equations (Eq. 1-3) will be rewritten as :

$$X(k+1) = G(X(k)),\ U(k),\ k) \tag{4}$$

The constraints on the initial and final states are :

$$V(1) = V_t \tag{5}$$

$$L(1) = L_t \tag{6}$$

$$V(N+1) = V_g \tag{7}$$

$$L(N+1) = L_s \tag{8}$$

where L_t is the longitude at the first apogee of the transfer orbit and L_s is the on-station longitude. The process studied will be called an N-stages process.

The constraints are : whatever $k \in \{1,\ldots,N\}$

$$V(k) \leq V(k+1) \tag{9}$$

which implies that the satellite velocity is always increasing

$$L(k)_{min} < L(k) < L(k)_{max} \quad \text{if } E(k) = 1 \tag{10}$$

This constraint is relative to visibility to ground station during the maneuvers.

The constraint on time interval between two apogee maneuvers is included implicitly in Eq. 1. As a matter of fact we see that if there is a burn at apogee k, E(k) = 1 this implies E(k+1) = 0, so there can't be a burn at apogee k+1. We notice also that if there is no burn at apogee k.

E(k) = 0 so we can have two possibilities at apogee k+1 : either a burn , either no burn according to the value of the decision variable C(k).

We give now the expression of the cost function :

$$J = \sum_{k=1}^{N} F(X(k),\ U(k),k) \tag{11}$$

with :

$$F(X(k),U(k),k)=(V_g-V(k)).\left(\frac{\Delta V(k)}{V_g-V(k)} -1+ \sqrt{\left(1-\frac{\Delta V(k)}{V_g-V(k)}\right)^2 +e_d^2\,\frac{\Delta V(k)}{V_g-V(k)}}\right) \tag{12}$$

Where c is a constant and e_d is the error in declination of thrust vector, this error is supposed to be a constant.

Eq. 12 gives the expression of the local cost of the maneuver at apogee k which is relative to the error in declination of thrust vector. The variation of the local cost (in m/s) in function of $V(k) + \Delta V(k)$ is shown in Figure 1.

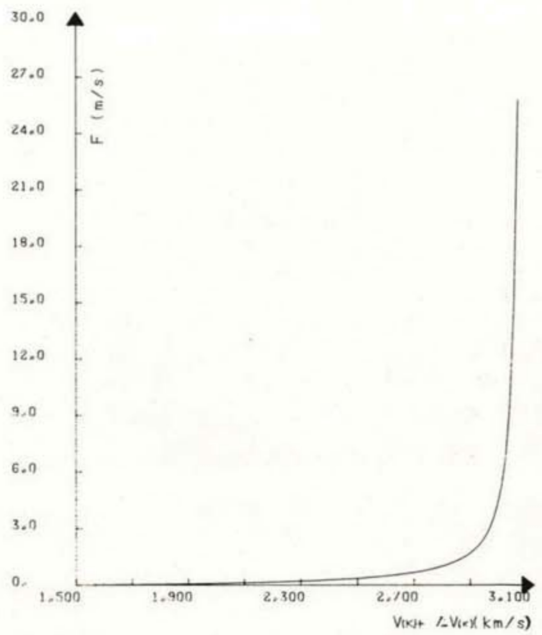

Figure 1. Cost of the impulse apogee maneuver
V_t = 1.597 km/s, V_g = 3.074 km/s,
$e_d = 1.74\ 10^{-2}$ rd.

We have chosen a dynamic programming method to solve the simplified orbit correction problem.

Dynamic programming is a global decision making procedure based upon the fundamental notion of feedback. The decision rules depend on the current state of the process. As an advantage we see that for a station acquisition problem dynamic programming can take into account thrust uncertainties, nevertheless this point is not discussed in this study. The main drawback of dynamic programming is the curse of dimensionality. The algorithm we propose is based on a backward dynamic programming procedure.

The problem we have to solve is :

$$\min_{U(1),U(2),\dots,U(N)} J \tag{13}$$

suject to the system equation (Eq. 4), the constraints on the initial and final states (Eqs. 5-8) and mission constraints (Eqs. 9-10).

This problem corresponds to the optimization of a multistage decision process. This class of problem is the kind to which dynamic programming applies. The basic dynamic programming approach to the above problem consists to define a minimum cost function I(X,k) as the minimum cost of going to the end of the process by using an admissible decision sequence from an arbitrary admissible state X and an arbitrary stage k with $1 \leqslant k \leqslant N$. So :

$$I(X,k) = \min_{U(k),U(k+1),\dots,U(N)} \left\{ \sum_{j=k}^{N} F(X(j),U(j),j) \right\} \tag{14}$$

with X(k) = X

Eq. 14 can also be written as the following functional equation :

$$I(X,k) = \min_{U(k)} \left\{ F(X,U,(k),k) + I(G(X,U(k),k),k+1) \right\} \tag{15}$$

The minimum cost from state X and stage k is obtained by minimizing the sum of the local cost F(X, U(k),k) plus the minimum cost from the resulting next state G(X, U(k),k) and stage k+1. Let I(X,k+1) be known for all admissible X, Eq. 15 gives an iterative relation for obtaining I(X,k) for all admissible X and for all k and in particular for k=1. We notice that by carrying the minimum cost function backward one stage at a time we will be able to compute the minimum cost at any state and stage. This recurrence relation which is the fundamental result of dynamic programming is an immediate consequence of Bellman's principle of optimality (ref. 4).

It is clear that the dynamic programming computational procedure can be implemented only for a finite number of states and decisions. So it is necessary to quantize state and decision variables. In order to cope with the curse of dimensionality the algorithm that we propose implements first a backward dynamic programming procedure for a nonfine quantization. Then the algorithm iterates around this solution via backward dynamic programming with a finer quantization (ref. 5).

For computational facilities the constraint on the final longitude (Eq. 8) has been eliminated by adding to the cost J the penalty $\Phi(X(N+1))$. The expression of the final cost is :

$$\Phi(X(N+1)) = C\,(L(N) - L_s)^2 \tag{16}$$

3. LOW THRUST MANEUVER OPTIMIZATION

In this study we have concentrated on a simplified low thrust injection problem. We have considered a two-axis orbit correction problem i.e. without inclination and we have neglected the terms relative to nonsphericity of the earth, sun and moon gravity, etc.

The system equations in earth-centred cartesian axes are :

$$\left\{\begin{aligned} \frac{dx}{dt} &= \overset{\circ}{x} \\ \frac{dy}{dt} &= \overset{\circ}{y} \\ \frac{d\overset{\circ}{x}}{dt} &= -\frac{\mu x}{r^3} + \varepsilon \frac{F_x}{M} \\ \frac{d\overset{\circ}{y}}{dt} &= -\frac{\mu y}{r^3} + \varepsilon \frac{F_y}{M} \\ \frac{dM}{dt} &= -\frac{\varepsilon F}{g_o I} \end{aligned}\right. \tag{17}$$

Where x and y are the coordinate of the satellite, μ is the earth's gravitational constant, r is the module of the geocentric position, ε is a parameter equal to 1 if there is a burn and equal to 0 if there is no burn, M is the satellite mass, F_x and F_y are the coordinates of the thrust vector F, g_o is the universal gravitational constant and I is the specific impulse of the apogee engine.

In order to apply a dynamic programming method to low thrust maneuver optimization we introduce from Eq. 17 the following discrete-time version of the system equations :

$$X(k+1) = G(X(k),\ U(k),k) \tag{18}$$

Where the burns are held to be discrete events, i.e. each maneuver corresponds to a stage. The components of the state vector X(k) are the orbit parameters of the satellite : semi major axis a, eccentricity e, argument of perigee w, the longitude at the apogee L and the satellite mass M. The components of the decision vector U(k) are : the true anomaly at start of burn v and the burning time t_b of the maneuver. Eq. 18 comes from the following computations.

Computation of the longitude, relative to the starting time of the maneuver from the knowledge of the longitude at the apogee and the true anomaly at start of burn. Change of coordinates from the orbit coordinates to the Cartesian coordinates. Numerical integration of the system equations relative to the burn. Change of coordinates from the Cartesian coordinates to the orbit coordinates and computation of the longitude at the apogee of the drift orbit relative to the next stage.

The constraint on the final state is :

$$X(N+1) = X_F \tag{19}$$

Where X_F is characterized by a semi major axis a_s equal to 42164.2 km an eccentricity equal to zero and an on-station longitude equal to L_s.

The constraint on the initial state is :

$$X(1) = X_I \quad (20)$$

Whatever the stage k the longitude of the satellite, $l(k)$, must satisfy the following relation during the burn in order to comply with the constraint of visibility to ground stations:

$$l(k) \in L_{ad}(k) \quad (21)$$

where $L_{ad}(k)$ is the segment : $]L_{min}(k), L_{max}(k)[$
The constraints on attitude determination and power generation imply that the angle Sun-Earth-Satellite, α , must satisfy :

$$\alpha_{min} < \alpha < \alpha_{max} \quad (22)$$

where α_{min} and α_{max} one some constants; The angle α verifies :

$$\alpha = W + v - \beta t \quad (23)$$

Where w is the argument of perigee, v is the true anomaly at start of the burn, β is the Earth's velocity and t the time variable.

The expression of the cost function is :

$$J = \sum_{k=1}^{N} F'(X(k), U(k),k) \quad (24)$$

with :

$$F'(X(k),U(k),k) = K_1.F(X(k),U(k),k)+K_2.\ \Delta V(k) \quad (25)$$

$F(X(k), U(k),k)$ satisfy Eq. 12 with :

$$e_d = \frac{K_3}{\sin\alpha}$$

K_1, K_2, K_3 are some constants and $\Delta V(k)$ is the velocity increment at stage k. Eq. 25 gives the expression of the cost relative to the kth maneuver. This cost is a combination of cost relative to error in declination of thurst and cost relative to consumption.

The low thrust optimization problem is :

$$\min_{U(1),U(2),\ldots,U(N)} J \quad (27)$$

suject to the following constraints : Eq. 18 - 22

Again for computational facilities the constraint on the final state has been removed by adding to the cost J the penalty $\Phi(X(N+1))$ with the following expression :

$$\Phi(X(N+1))=C_1(r_p(N+1)-a_s)^2+C_2(L(N+1)-L_s)^2 \quad (28)$$

where C_1 and C_2 are some constants and $r_p(N+1)$ is the perigee radius at the N+1-th stage. $r_p(N+1)$ verifies :

$$r_p(N+1) = a(N+1)\ (1 - e(N+1)) \quad (29)$$

Notice that we are considering an N-stage decision process. We can apply dynamic programming to the low thrust optimization problem. Nevertheless the main drawback of this method remains the curse of dimensionality because of the dimension of this problem.

In order to cope with this drawback we propose the following iterative procedure :

(a) Compute an initial low thrust strategy from the optimal solution of the impulse maneuver problem. The burn are deduced from the velocity increments. The true anomaly at start of burn gives symmetrical burns around the apogee.

(b) Quantize a domain around the previous solution and apply a backward dynamic programming procedure on this domain.

(c) If the solution is obtained with a given precision on the final state:
then stop
else modify the quantization increments or move the domain of quantization and go to step (b)

4. EXPERIMENTAL RESULTS

The research was supported by a contract with the Centre National d'Etudes Spatiales, Toulouse. C.N.E.S. submitted us the problem of the orbit correction of a TDF1 type geostationary satellite, with 400 N engine. The initial state of the satellite is :

semi major axis	a	= 24371,1km
eccentricity	e	= 0,73
argument of perigee	w	= 205°
longitude at the first apogee	L(1)	= 90°
satellite mass	M	= 2000 kg

The on-station longitude required is 19° west.

The problem studied is highly constrained. There are limitations on the total time for station acquisition, time interval between apogee maneuvers, visibility to ground stations : the longitude of the satellite during the maneuvers must be included within a range of 60° west, 60° East. There is also a limitation on location of the spacecraft in the orbit with respect to the sun : the angle sun - Earth - Satellite must be superior to 15° and inferior to 45°.

The experimental results for various horizons of optimization are presented in table 1, 2 and 3.

	impulsive problem		low thrust problem		
k	L(k) (°W)	ΔV(k) (m/s)	v (o)	t_b (s)	α (o)
4	24.6	594.9	176.6	2670	20.4
6	59.8	820,7	175.1	2968	17.7
8	19	61.6	180.1	189	20.9

Table 1. N = 8

	impulsive problem		low thrust problem		
k	L(k) (°W)	ΔV(k) (m/s)	v (o)	t_b (s)	α (o)
4	24.6	1028.4	174.6	4349	18.2
7	59	395.6	177.6	1312	19
9	24	47.2	179.9	145	19.5
11	19	6	183.1	24	19.8

Table 2. N = 11

	impulsive problem		low thrust problem		
k	L(k) (°W)	Δ V(k) (m/s)	v (o)	t_b (s)	α (o)
4	24,6	562	177	2562	20.4
6	54	573,5	176.6	2160	19.2
10	56	292,8	178.2	953	18
12	24	42,8	180	132	17.7
14	19	6	183	19	16.4

Table 3. N = 14

where α is the angle Sun - Earth - Satellite at start of burn and for a thrust vector perpendicular to the vector Earth - Satellite.

Table 4 gives some details on the solutions of the low thrust problems :

	N		
	8	11	14
D $\simeq$ (days)	4.5	7.5	9.5
r(N+1) (km)	42164	42163	42163
e	6.10^{-4}	1.10^{-3}	$3.3.10^{-4}$
L(N+1) (°W)	13	19	19
M(N+1) (kg)	1218	1217.6	1218.3
C (m/s)	12.2	7.6	4

where D is the duration of station acquisition maneuvers and C is the cost of the errors in declination of thurst.

The experiments have been carried out on a VAX 11/780.

The method that we present permits in a relatively brief time to find a global optimum of the impulsive maneuver optimization problem for a given horizon and with constraints of visibility to ground station. Afterwards the programmer makes some choices on the various objectives of the mission : precision on the final state, or error in attitude determination before the implementation of the low thrust procedure. The latter implementation is not time expensive: 2 or 3 hours on a VAX 11/780.

The multi-burn strategy for N = 14 is shown in Figure 2 with geographic coordinates. The burns are numbered and represented in thick lines.

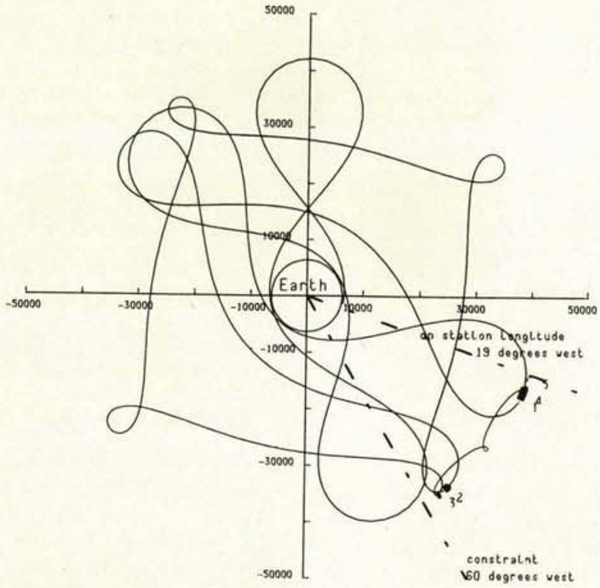

Figure 2

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