INTERPOLATION OF PERFORMANCE PARAMETERS FOR CATALYTIC HYDRAZINE THRUSTERS*

J Kollien

MBB/ERNO Bremen
Germany

J Weiss

Hochschule Bremen
Germany

ABSTRACT/RESUME

The evaluation of an algorithm is described, which allows calculation of time dependent thrust force shapes over a wide range of thruster pulse modes and fuel supply pressures. Figures of the algorithm parameters are given w.r.t. a 2 N Hydrazine thruster. Main results of an accuracy check comparing measured and calculated performance data are presented.

Keywords: Thruster Model, Thruster Performance Data Interpolation, Hydrazine Thruster

1. INTRODUCTION

For spacecraft attitude and orbit control systems Hydrazine is often used as propellant. In spite of its smaller specific impulse compared to bi-propellants, it has some definite advantages:

- it establishes less complex systems, requiring less hardware, and
- it is more compatible to materials than, for example, NTO;
- qualified thrusters with thrust levels down to 0.5 N are available;
- reproduceable minimum impulse bits down to 0.005 N·s can be achieved.

These four highlights, and others, result in a frequent use of Hydrazine systems, for example, for the missions of INTELSAT, OTS, ECS, METEOSAT and GIOTTO in the past, and for HIPPARCOS, EURECA and ULLYSSES (ISPM) in the future.

In order to make real use of the highly sophisticated methods and procedures available today for spacecraft attitude and orbit control, and in order to complete these methods and procedures, it is necessary to have informations on the thruster performance at all possible system conditions and especially during pulsed mode firings. Usually these informations are derived during acceptance test firings of the thruster. However, practical reasons exist which limit the quantity of such tests: to be mentioned are the costs and the wear-off, going along with the firings. So, normally informations for 3 fuel supply pressures and a number of 2 or 3 duty cycles are available.

Data on operational modes differing from the tested modes can be derived to a certain extent by plain interpolation and extrapolation of given performance data, but often the estimated figures are of unsatisfactory accuracy. So a mathematical thruster model was developed which delivers accurate informations on the thruster performance over a wide range of possible firing modes.

The physical and mathematical background of this model is briefly described in the following, the model parameter evaluation procedure is demonstrated, and some examples for the exactness of the model are given.

2. DESCRIPTION OF THE MODEL

In principle, two major types of thruster models are thinkable:

- a physical or analytical model, based on a detailed description of the chemical and physical phenomena occuring during thruster firing
- a mathematical model which correlates the results of a lot of different thruster firing experiments (interpolation model).

*) The original work was done as ESA study. A detailed description of the results is given in Ref. 1.

In consideration of a large amount of existing experimental thruster test data, the second type of model was chosen for further analysis. Moreover, it was expected that this model would bring with defendable efforts better results than a physical model.

The available test data were consisting of measurements, giving the thrust force versus time for various kinds of thruster pulses and pulse trains, as well as for continuous firing conditions, related to a Hydrazine thruster with 2 N nominal thrust. The time increments of the measurements were 1 ms respectively 2 ms for pulses with longer thruster valve on-times. By this a very precise description of the course of the thrust force during the pulse is given. So it was obvious to define a force-time-function to be the basis for the thruster performance calculation model, see Fig. 1.

Force-Time-Function

mathematically:

$F = F\,(t_{on}, t_{off}, i, p, t)$

Independent variables are

t	time, and
t_{on}/t_{off}	thruster valve on/off time
i	pulse number
p	fuel supply pressure

graphically:

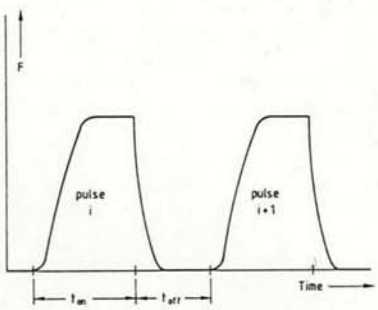

Fig. 1: Force-Time-Function

From this function, the important quantities of absolute impulse bit, centroid time and mean thrust force can be derived by integration. The rotational impulse bit and centroid time can be easily calculated, too, for any given satellite spin rate. With knowledge of the mass flow rate per pulse, even the specific impulse can be derived from the force-time-function.

3. EVALUATION OF THE MODEL PARAMETERS

An example of a measured pulse is plotted in fig. 2. It indicates that the pulse consists of two substantial parts, characterized by an exponential raise with following plateau, and a decay part likewise similar to an exponential one. So the function describing the force-time relation should include some exponential elements.

A simple physical reflection confirms this. A mass balance for the thruster gives

$$\frac{d}{dt}\,(y) = \dot{m}_{in} - \dot{m}_{out} \qquad (1)$$

with $\frac{d}{dt}\,(y)$ time derivative of mass content in the thruster

$\dot{m}_{in}/\dot{m}_{out}$ mass flow entering/leaving the thruster.

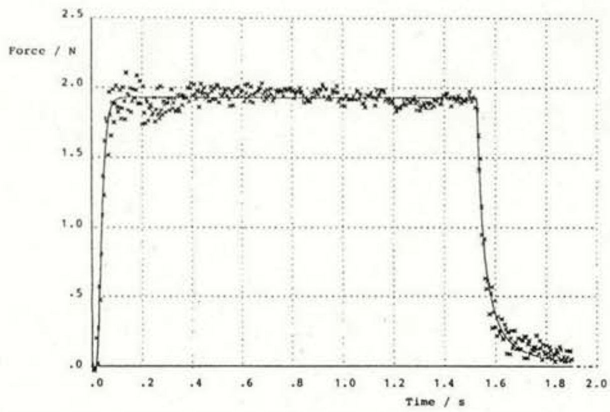

Fig. 2: Example of a measured pulse (crosses) together with fitted curve.

Looking at the thrust raise and decay part of the pulse, and assuming for a first approximation a constant $\dot{m}_{in} = a$ and $\dot{m}_{out}$ proportional to the mass content y, namely

$$\dot{m}_{out} = b \cdot y \qquad (2)$$

with b = constant, one gets with the initial condition y = 0 at t = 0:

$$y\,(t) = \frac{a}{b} \cdot (1 - e^{-\,b\,\cdot\,t}) \qquad (3)$$

The thrust force may be set proportionally to the mass flow leaving the thruster,

$$F \sim \dot{m}_{out} \qquad (4)$$

or, with eqs. 2 and 3 this results in

$$F\,(t) = F_{\infty} \cdot (1 - e^{-\,b\,\cdot\,t}). \qquad (5)$$

The decay part of the pulse is initiated by letting $\dot{m}_{in}$ in eq. 1 become zero.

This results in

$$y\,(t) = y_1 \cdot e^{-b\cdot(t\,-\,t_1)} \quad \text{with} \qquad (6)$$

$$y_1 \;=\; \frac{a}{b}\cdot(1 - e^{-b\,\cdot\,t_1}) \quad \text{and}$$

t_1 being the time when the valve shuts.

From this follows

$$F\,(t) = F^* \cdot e^{-b\,\cdot\,(t\,-\,t_1)} \quad \text{with} \qquad (7)$$

$$F^* \;=\; F_\infty \cdot (1 - e^{-b\,\cdot\,t_1}). \qquad (8)$$

To be correct, it has to be mentioned that the real pulse begins at t = 0 having a slope of d/dt(F) = 0, whereas eq. 5 gives d/dt(F) = $F_\infty \cdot b > 0$. The realistic behaviour can be copied in the mathematical force-time-function by changing eq. 2 to

$$\dot{m}_{out} = b \cdot y^q \quad \text{with } q > 1, \qquad (2a)$$

leading, for example, for q = 2 to equations for the thrust force, which contain the hyperbolic tangent. However, it turned out during the study work, that the models with q > 1 bring no or only very small improvement in accuracy, but complicate the calculations and increase the calculation time. So they were not used as reference. Instead of that, eqs. 5 and 8 were extended as follows.

The measurements show a time delay between electrical signal 'valve on' and the beginning of the pulse. In the same way there is a time delay between electrical signal 'valve shut' and the beginning of the thrust decay. These two delays are introduced in the formulae with help of the time increments τ and τ^* . In addition it was found, that the thrust decay can be better described by two exponential functions, bringing a second time constant b* and a ratio of decay amplitudes, γ .

The final force-time-function then is

$$F(t) = \begin{cases} 0 & \text{for } t \le \tau \\ F_\infty\cdot(1 - e^{-b(t-\tau)}) & \text{for } \tau < t \le t_{on}+\tau^* \\ F_\infty\cdot(1 - e^{-b(t_{on}+\tau^*-\tau)})\cdot[(1-\gamma)\cdot e^{-b(t-t_{on}-\tau^*)} \\ \quad +\gamma\cdot e^{-b^*(t-t_{on}-\tau^*)}] & \text{for } t_{on}+\tau^* < t \end{cases} \qquad (9)$$

with the pulse form model parameters

F_∞ force at steady state firing

b, b* time constants for thrust raise/decay

τ, τ^* time delay for valve opening/closure

γ ratio of decay amplitudes.

t_{on} is the valve open time, t is the time variable, starting with t = 0 when the thruster valve is opened.

4. DETERMINATION OF MODEL PARAMETERS

The 6 parameters in eq. 9 were calculated pulse by pulse for the variety of 14 pulse modes sketched in fig. 3. Additionally steady-state firing data for thruster valve on-times up to 200 s were available. Thereby the number of pulses in a pulse train was 120 for short (t_{on} + t_{off})-times and 75 for long (t_{on} + t_{off})-times. Fuel supply pressures belonging to the modes were 22, 10 and 5.5 bar. So in total a number of approximately 5000 different pulses was analysed. Considering a mean number of thrust force measurements per pulse of about 500, a number of 2.5 million measurement data was processed.

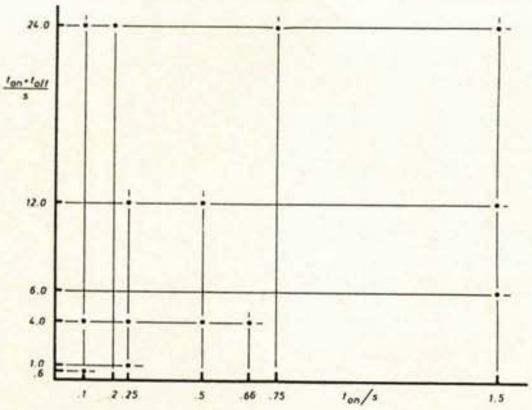

Fig. 3: Duty cycles of available test data

The parameter estimation procedure was based on a common minimum search problem. Objective is, to find the thruster model function parameters such, that the squares of the differences between measured and calculated data are minimized. Because the objective function is not linear, the minimum search was performed with help of a standard routine for constrained non-linear optimization, written by Bartholomew-Biggs (Ref. 2).

Result was a set of 6 parameter figures for each of the 5000 pulses.

In a second step these parameters were correlated w.r.t. the pulse number within the pulse train, they belong to.

When doing this, the parameters F_∞, b and τ show a trend to saturation with increasing pulse numbers, so that they can be described by an expression of the form

$$U = U_1 + U_2 \cdot e^{-\beta\cdot(i\,-\,1)} \quad \text{with} \qquad (10)$$

i pulse number and

U_1, U_2, β constant figures for one pulse train

The parameters b*, τ * and γ show a more or less random behaviour with respect to the pulse number, so that for further analysis their mean values were taken.

The pulse form model parameters in eq. 9 are then defined for any given pulse i in a pulse train by the following 'pulse-train-model':

$$F = F_1 + F_2 \cdot e^{-\beta \cdot (i - 1)} \tag{11}$$

$$\tau = \tau_1 + \tau_2 \cdot e^{-\beta \cdot (i - 1)}$$

$$b = b_1 + b_2 \cdot e^{-\beta \cdot (i - 1)}$$

$$\tau^* = \overline{\tau^*};\ b^* = \overline{b^*};\ \gamma = \overline{\gamma}$$

It turned out that the individual time constants β for F_∞, b, and τ were not very different from each other, so that they were replaced by a common time constant. The parameters in this model were calculated for each given pulse train from the pulse-form-model parameters again with help of the procedure from Bartholomew-Biggs. This resulted in 42 sets of parameter values, namely one for each duty cycle - pressure combination.

With the 10 × 42 pulse train model parameters a last interpolation was made w.r.t. valve open/close time t_{on}/t_{off}, and w.r.t. the fuel supply pressure p. Several interpolation formulae were tried, and it was found, that a good fit is given by the following three:

For F_1 and F_2 in eq. 11:

$$F_{1,2} = \frac{p}{10} \cdot \left(a_{1,2} + \frac{p}{10}\left(b_{1,2} + \frac{p}{10} c_{1,2}\right)\right) \cdot \left(1 + \frac{d_{1,2}}{t_{on} + 0.01}\right) \cdot \left(1 + \frac{e_{1,2}}{t_{on} + t_{off}}\right) \cdot \left(1 + \frac{f_{1,2} \cdot \sqrt{t_{off}}}{t_{on}\,(6 + t_{off})^2 \sqrt{p}}\right) \tag{12}$$

For τ_1 and τ_2 in eq. 11:

$$\tau_{1,2} = \left(a_{1,2} + \frac{p}{10}\left(b_{1,2} + \frac{p}{10} c_{1,2}\right)\right) \cdot \left(1 + d_{1,2}\sqrt{t_{on}}\right) \tag{13}$$

For $\overline{\tau^*}$, b_1, b_2, $\overline{b^*}$, $\overline{\gamma}$ and β :

$$z = a_z + b_z \cdot \sqrt{t_{on}} + c_z \cdot p \tag{14}$$

where z stands representative for $\overline{\tau^*}$, b_1, b_2, $\overline{b^*}$, $\overline{\gamma}$, β .

These three relations form the initial equations for the thruster performance calculation algorithm. The parameter figures were found again with help of a minimum search, they are listed in Table 1.

To use the algorithm in order to calculate the thrust force shape of a definite pulse characterized by a certain t_{on}, t_{off}, pressure and pulse number,

firstly, calculate the parameters of the pulse-train-model applying eqs. 12 to 14

secondly, calculate the pulse-form-model parameters applying eq. 11

thirdly, calculate the pulse shape applying eq. 9.

5. ACCURACY OF THE THRUSTER PERFORMANCE CALCULATION ALGORITHM

The accuracy of the algorithm described above was tested in two ways.

Firstly, the mean force per pulse and the integral quantities impulse bit (I_{bit}) and centroid time (t_c) and rotational impulse bit and centroid time for a spin rate of 5 RPM were calculated directly from the measured thrust force data and then compared to data calculated with the thruster model algorithm.

The absolute mean deviations considering all measured pulses were

for I_{bit}	0.0074 N · s
t_c	0.0081 s
rotational I_{bit}	0.0073 N · s
rotational t_c	0.0080 s
mean force	0.0309 N.

A second accuracy check was performed using acceptance test data of 13 ULLYSSES (ISPM) 2 N thrusters. Available were computer tapes with results for duty cycles with t_{on}/t_{off}-times 0.1/48 s, 0.75/23.25 s and 1.5/10.5 s.

Hereby for nearly all compared pulses higher mean thrust forces were calculated than measured. The absolute mean deviation was 0.042 N considering all 13 thrusters. The corresponding abs. mean deviation in centroid time was 0.0064 s. These deviations could be reduced by introducing two constant but thruster specific figures, namely a factor to be multiplied with F_1 in eq. 11 and a term to be added to τ_1 in eq. 11. Factor and term were derived such, that the absolute mean deviations of mean thrust force and centroid time between measurements and calculations for the modes with 1.5 s on-time were minimized.

To calculate the the pulse-train parameter ...	use equation ...	with the following parameter figures for ...					
		a	b	c	d	e	f
F_1	(12)	1.20591	-.16986	.1660	-.02631	.05154	4.17243
F_2	(12)	-.0698	-.0501	.0147	-.0580	.2162	71.49
τ_1	(13)	.2221	.008285	-.002708	-.1692		
τ_2	(13)	.0064	-.00638	-.00165	5.268		
$\overline{\tau^*}$	(14)	.03831	.00725	-.00030			
b_1	(14)	44.08	-15.85	.809			
b_2	(14)	14.2	4.49	-1.4			
$\overline{b^*}$	(14)	0.0	26.29	.996			
$\overline{\gamma}$	(14)	.274	.303	0.0			
β	(14)	0.0	.164	.0073			

Table 1: Figures of parameters in eqs. 12, 13 and 14

In this way the thruster performance calculation model was adjusted to each of the treated ISPM thrusters leading to the following abs. mean deviations:

impulse bit	0.0228 N · s
mean force	0.0235 N
centroid time	0.0059 s.

ACKNOWLEDGEMENT

The work done was founded by the European Space Agency. The authors want to thank especially Mr. L. Fraiture (ESOC, Darmstadt) and Mr. B. Berry (ESTEC, Noordwijk) for their assistance and interest, which contributed a lot to the study outcome.

REFERENCES

1. Baldewein S, Kollien J, Weiß J, Study of Calibration Interpolation of Catalytic Hydrazine Thrusters in Pulsed Mode, Final Report, parts 1, 2, 3; ESOC Contract No. 5879/84/D/IM(SC), 1984/1985.

2. Hock W, Schnittkowski K, A Comparative Performance Evaluation of 27 Non-Linear Programming Codes; Computing 30, 335 - 358 (1983)