CONSTELLATION DESIGN OPTIMIZATION WITH A DOP BASED CRITERION

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Abstract

The main aim of this paper is to present a general method to optimize a constellation geometry along with a DOP (Dilution Of Precision) based criterion. The constellations we are interested in are homogeneous, uniform and phased, like the Walker constellations, but we will also introduce a new configuration that is a combination of Walker constellations to improve the performances. We present two methods to solve this problem. The first one deals with the mean and the worst DOPs of a constellation with the help of a discretization of the service area and the simulation time. The second method focus directly and precisely on the worst DOPs, to emphasize the weaknesses of a constellation geometry. These two methods prove to be equally useful and complementary. Finally, we will apply these algorithms on two problems. The first one is the classical MEO constellation for global coverage, and the second, an hybrid LEO+GSO constellation intended to provide a navigation service over Europe.

Key words: optimization, satellite constellation, Walker constellation, GDOP, positioning, navigation.

Introduction

Satellite constellations are fundamentally linked to the future of the space development. The miniaturization of satellite components along with a broader and cheaper access to space allow the satellite constellations to be considered as viable and profitable alternatives to more classical space systems. This is the case of the telecommunication industry, as constellation projects are set up to provide new mobile phone and multimedia communication. But, this is even more sensitive for a positioning and navigation service, where the specific nature of the service imposes a multi-satellite visibility for the users (as opposed to a communication service, where one visible satellite is often sufficient). Hence there is a need for an efficient optimization tool to find the right number of satellites, and their associated orbital parameters, to fulfil the mission requirements.

Constellation Design

The optimization of a constellation is not an easy task. A lot of factors (technological feasibility, reliability, efficiency, costs, etc.) have to be taken into account, and many are in opposition. In initial studies, we often restrict ourselves to the mission performance objectives. They allow us to define some basic concepts or solutions, that could be further used as starting points for more complex studies. Even if they do not necessarily fulfil all the mission requirements, constraints and objectives, they give very good hints to design the final constellation geometry.

Besides, in this paper we will focus on an initial constellation design that rely only on the performances and more precisely, in this case, on the positioning errors. Even if they could also be considered all together (with a multiple objective optimization, for example), the other parameters are lay aside for further refinements.

Many authors have addressed the constellation design problem before, but it is not our objective to present in this paper a survey of all the proposed solutions. Usually, they propose a solution to the classical problem of a continuous partial or global Earth coverage with one or more satellites. Some solutions are almost completely analytical (for example, Rider[1] and his streets of coverage), but in general, they need a numerical tuning of the constellation, like the elegant solution proposed by Walker[2,3,4] that we will use as a basic design for our optimization process. A performance comparison of both approaches has been presented by Lang[5].

We preferred the Walker constellations because they exhibit some very interesting properties. In a Walker constellation, all the satellites are regularly distributed on similar and phased orbital planes. So we need only 5 parameters to characterize the constellation:

T : number of satellites
P : number of orbital planes
F : phasing factor
a : semimajor axis
i : inclination

As they are homogeneous and uniform, the Walker constellations provide a periodic but regular, global or zonal coverage that can be quickly estimated. Moreover, circular orbits with identical inclinations favor a similar behavior of the satellites under orbital perturbations and minimize the constellation dynamical distortions.

But, even if many Walker constellations provide very good coverage, we will introduce new constellation configurations that still benefits from the nice features of the Walker constellations, but also add more flexibility to the coverage. We will name them multi-Walker constellations because they are combinations of basic Walker constellations. In general, they will share the same semimajor axis and inclination (to preserve the dynamical behavior), but not necessarily similar T, P, F parameters. To position every basic constellations we need two more parameters that set the initial satellite location:

Ω_i : initial longitude of basic constellation *i*
α_i : initial latitude argument of basic constellation *i*

As we will show in the examples, we can often improve the performances of a good Walker constellation if we optimize a translation of this initial constellation in a more flexible multi-Walker (every Walker can be viewed as a specific combination of smaller identical Walker constellations). So, the combination (or splitting) rules of Walker constellations can be defined below:

U : pattern unit (= 360^o/T)
M : number of satellites per plane (= T/P)
n : factor of combination/splitting

Modification of the number of orbital planes:

combination	splitting
T’ = n T	T” = T / n
P’ = n P	P” = P / n
U’ = U / n	U” = n U
M’ = M	M” = M
F’ = F + (0 .. n-1) P	F” = modulo(F, P”)

Modification of the number of satellites per plane:

combination	splitting
T’ = n T	T” = T / n
P’ = P	P” = P
U’ = U / n	U” = n U
M’ = n M	M” = M / n
F’ = modulo(n F, P’)	F” = F / n

Optimization Criterion

Usually, the constellations are selected with the help of an optimization criterion based on the satellite visibility from the users. This criterion is simple and easily implemented, and it allows a quick overview of the candidate constellations. It is very well suited for communication constellations, that expect a good zonal or global coverage. But even if it can provide good insights, a coverage criterion is not sufficient to guarantee a good navigation service. Obviously, it can verify the 4-fold coverage we need (it is well known that a minimum of four visible satellites is needed to compute position, velocity and time information), but it provides no indications about the geometrical layout of the satellites, an important contributor to the positioning precision.

Hence, we propose to optimize the constellations directly with a DOP (Dilution Of Precision) based criterion even if it proves to be more complex and more computer intensive. This criterion is defined as the ratio of the rms position error of the user to the rms ranging error from the satellites (assuming gaussian, identically distributed and uncorrelated pseudorange errors). The DOPs are listed by types:[6]

Geometric DOP: $GDOP = \frac{1}{\sigma}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_t^2}$

Position DOP: $PDOP = \frac{1}{\sigma}\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}$

Horizontal DOP: $HDOP = \frac{1}{\sigma}\sqrt{\sigma_x^2 + \sigma_y^2}$

Vertical DOP: $VDOP = \frac{\sigma_z}{\sigma}$

Time DOP: $TDOP = \frac{\sigma_t}{\sigma}$

σ : rms pseudorange error of a satellite
σ_x, σ_y, σ_z : rms errors of the user position
σ_t : user clock bias error

As stated before, to evaluate the DOPs we need at least four visible satellites. But of course, if more satellites are available all the pseudorange measurements will be used to compute a user location. However, it should be pointed out that the DOPs do not fit exactly with the real errors in the estimated user position because the pseudorange errors may be quite different from one satellite to another (for example, these errors often vary as a function of the elevation angle because of the atmospheric perturbations). But they still provide a useful measure of performance that takes into account the geometry of the satellites relative to the users.

Optimization of a Constellation

The global optimization of a constellation geometry is a very complex and difficult task because it means solving a nonlinear problem with mixed integer and continuous optimization variables. An examination of the Walker constellations shows that small variations of the integer parameters T, P and F often translate into big performance gaps without any workable logic. This is especially true of the phasing factor F. Its setting is extremely sensitive and can exhibit very bad or very good performances, even if the constellations have the same number of satellites and orbital planes.

To overcome these concerns, we could use a global optimization algorithm that indiscriminately deals with integer or real variables, like the genetic algorithms for example. But in this paper, we propose another approach that can be used with profit if the set of candidate constellations is not too wide. It is based on a multisteps strategy that first begins with an exhaustive scanning of the candidate constellations to detect the most promising configurations. Afterwards, these configurations can be processed through a more complete and intensive optimization, that could eventually involve a split of the initial Walker constellation. This optimization strategy is developed below:

1. Evaluation of all Walker constellations within a bounded range of satellites (with every admissible orbital planes and phasing factors, and taking into account nominal semimajor axis and inclination) and selection of the most promising constellations.
2. Optimization of the semimajor axis and the inclination.
3. To improve the performances, splitting of the best Walker into multi-Walker configurations, and optimization of the relative positioning parameters Ω_i and a_i.

The optimization process can be stopped at any stage if the performances are satisfactory. But, a complete processing can provide very effective solutions.

In the previous section, we introduced the DOPs, that we intend to compute to measure the performances of the constellations. But we have to define more precisely the performance criterion. In fact, we propose two implementations that are complementary on several aspects. The first implementation is based on a discrete computation of the DOPs, but the second one uses a continuous formulation.

First method: discrete evaluation

We could use many types of performance criterion. For example, we could evaluate the percentage of time when the DOPs remain below specific thresholds[7]. At first, we rather choose to measure the mean DOP values. This criterion can be written as a relation that rely on a discretization of the simulation time and the service area:

$$J^{mean} = \sum_{x=G,P,H,V,T} c_x \left(\frac{1}{K} \sum_{k=1}^{K} \left(\sum_{u=1}^{U} c_u \cdot xDOP_{u,k} \right) \right)$$

c_x : weighting coefficient of xDOP
c_u : weighting coefficient of user u
U : number of users
K : number of time steps

The simulation time is divided in K equal steps (time interval Δt). Typically, the total simulation time should be as long as the geometrical repetitivity period of the constellation. The service area targeted by the mission is also subdivided in several distinct users. The nature of this subdivision is not so important. We could use indiscriminately a classical regular tessellation or a more sophisticated selection of the user locations that is better related to the mission requirements. The essential point is to simulate a good coverage of the service area. Besides, the coefficients c_u are usually used to notify the relative surface area associated with every user, but they could also emphasize the critical users.

As defined, the performance criterion is a weighted summation of all DOPs. But in general, it is sufficient to evaluate only one or two DOPs, depending on the mission. For example, terrestrial navigation needs essentially a good HDOP and airplane landing relies also on VDOP. So, with this formulation it is possible to combine many DOPs or simply focus on the more general GDOP or PDOP.

The computation of the DOPs requires many parameters (to locate the satellites and the user):

$$xDOP_{u,k} = f(T, P, F, a, i, u, k)$$

But as many parameters are already fixed, this relation is often reduced to the two discrete variables:

$$xDOP_{u,k} = f(u,k)$$

However, to evaluate a multi-Walker constellation we must take into account additional parameters to define the basic constellations *i*, and many of them could end as optimization variables if they are not fixed (Ω_i and α_i for example):

$$xDOP_{u,k} = f(T_i, P_i, F_i, a_i, i_i, \Omega_i, \alpha_i, ..., u, k)$$

The mean DOP values are a good indicator of the global behavior of a constellation, but they cannot guarantee that the DOPs do not sometimes reach excessive values (an important requirement for many missions). So, we could compute the worst DOP values for each user instead of the mean value[8], or we can also define a performance criterion that keeps only the worst DOPs of the entire service area:

$$J_1^{\max} = \sum_{x=G,P,H,V,T} c_x \left(\max_{k=1,K} \left(\max_{u=1,U} xDOP_{u,k} \right) \right)$$

As we use discretizations, we must also introduce the problem of the precision along with the computation time. They are closely tied together. It is important to select the discrete intervals according to the required precision, but not to the detriment of an exorbitant computation time, especially if we have a lot of constellations to evaluate. In this case, a continuous formulation (that avoids discretization) could be used advantageously.

Second method: continuous evaluation

The second solution is to compute the DOP values as a function of three continuous variables: the user coordinates in a geocentric referential on the Earth surface, and the simulation time:

$$xDOP = f(\Omega_u, \lambda_u, t)$$

Ω_u : user longitude
λ_u : user latitude
t : simulation time

The simulation time t is very important because it determines exactly the locations of the satellites viewed by the user. Moreover, discretization of the Earth surface is replaced by the user exact coordinates (Ω_u and λ_u), and that means a more precise evaluation of the worst DOPs. It should be noticed that the DOP functions are continuous and regular for the all the points that share the same set of visible satellites. With this new formulation, evaluate a constellation is equivalent to solve the following maximization problem:

$$J_2^{\max} = \sum_{x=G,P,H,V,T} c_x \left(\max_{\Omega_u \in D_\Omega, \lambda_u \in D_\lambda, t \in D_t} xDOP(\Omega_u, \lambda_u, t) \right)$$

The Earth surface to explore is bounded in longitude and latitude (D_Ω and D_λ), as well as the simulation time (D_t). Solving this maximization problem will provide a precise information on the worst DOP values of the candidate constellation (but it provides no clues about the mean DOP values). However, this problem is highly nonlinear and we need efficient and robust optimization algorithms to solve it. It is a problem perfectly suited for direct optimization methods, that do not rely on a derivative function of the performance index. To compute the results presented in this paper, we have selected a nonlinear simplex (Nelder and Mead[9]) with a multistart technique. If the number of starting points is sufficiently large, we can hope to detect the global optimum (the worst constellation's DOPs), but without any mathematical certitude unfortunately. Anyway, the DOPs computed with this method should be at least as worst as the ones obtained with the previous discrete problem. Therefore, this second solution is a good indication of the precision of the discrete solution and it proves to be an effective complement.

Applications

MEO constellations

At first, we will test the proposed optimization methodology on the classical MEO navigation constellation for global coverage. GPS and GLONASS constellations fall into this category and they have the great merit to work for many years. So we will use them as a benchmark to compare the performances of other constellations. To model the GPS constellation, we used the 24-satellite configuration presented by Spilker[6] (with a semimajor axis of 26561.75 km and an inclination of 55 degrees). GLONASS is simply a Walker 24/3/1 with a semimajor axis of 25478 km and an inclination of 64.8 degrees.

According to the first step of the optimization process, we computed mean and worst GDOPs for all Walker constellations within a 20 to 36 satellites range (with GPS semimajor axis and inclination). For calculation purposes, we choose a relatively fine tessellation of 2 degrees in longitude and latitude to model the global coverage, and a time step of 1 degree in anomaly.

As expected, we can see on the first figure that the mean GDOP of the best constellations decreases regularly with the increasing number of satellites. We can verify that GPS and GLONASS are performing well in the 24-satellite category (GPS is even slightly better than the best Walker constellation, a W24/6/1). However, the results are far less regular with the worst GDOPs (figure 2), and the most performing constellations are not necessarily the same than before. Besides, GPS and GLONASS exhibits very bad GDOPs for some latitudes (we must add that GPS has not been optimized according to this factor, but in relation to satellite failures). Moreover, in this case we do not see a significant improvement with more than 30 satellites.

In table 1, we summarized the performances of the most interesting constellations (mean and worst GDOPs together). The results obtained with the second method of evaluation of the worst DOPs are very close to the ones computed with the discrete method. Therefore, we can conclude that our discretization was fine enough.

The mean GDOP is function of the latitude, so at figure 3 we have drawn its evolution for the selected constellations. The more erratic behavior of the worst GDOPs is also shown at figure 4, where we clearly see the weakness of GLONASS around 27 degrees of latitude.

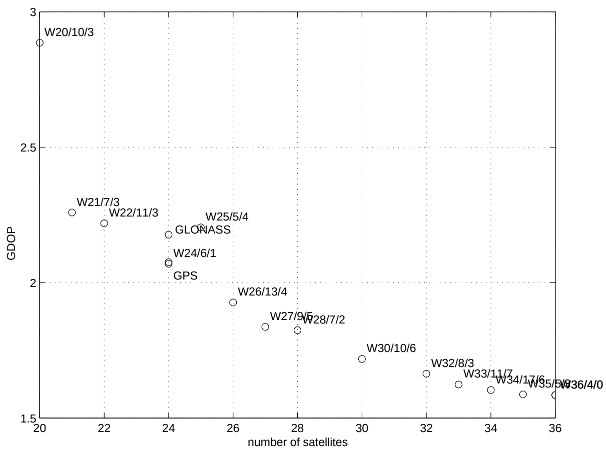

Figure 1: Walker constellations with best mean GDOP

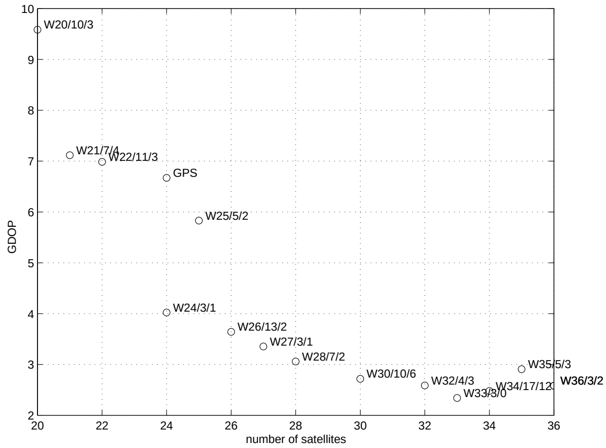

Figure 2: Walker constellations with best worst GDOP

Table 1 : Performances of MEO constellations

constellation	GDOP		
	mean	worst (1)	worst (2)
W22/11/3	2.22	6.98	7.02
W24/3/1	2.13	4.02	4.02
GLONASS	2.18	45.9	49.2
GPS	2.08	6.67	6.69
2 x W12/3/1	2.13	3.48	4.04
W26/13/2	2.01	3.64	3.64
W28/7/2	1.82	3.06	3.07
W30/10/6	1.72	2.72	2.76

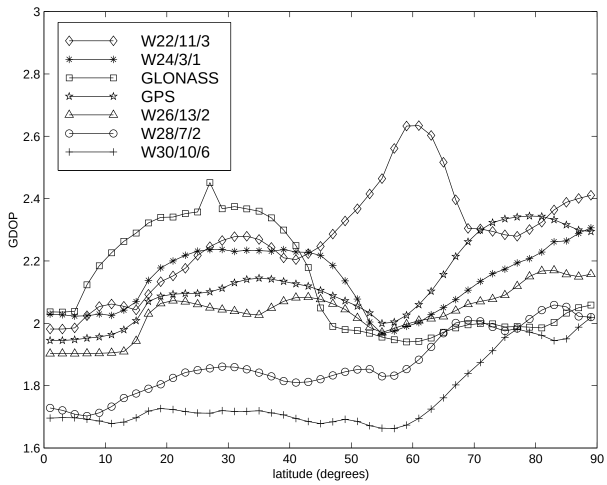

Figure 3: Mean GDOP evolution with latitude

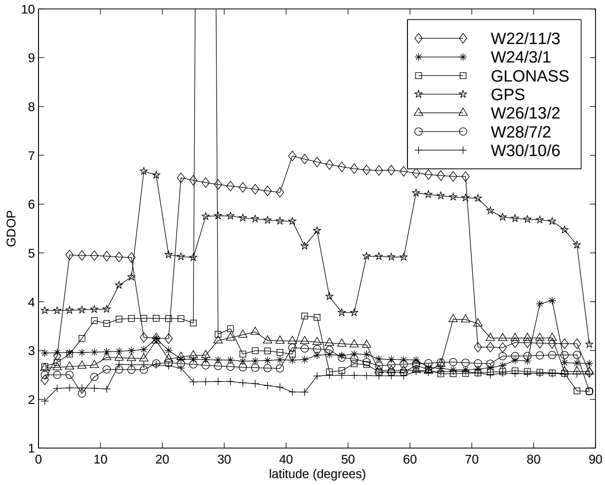

Figure 4: Worst GDOP evolution with latitude

According to the last step of the optimization process, we will also try to improve the performances with splitting and optimizing a good Walker constellation. So, we have selected the Walker 24/3/1, that has very good mean and worst GDOPs. This constellation has been split in two identical W12/3/1, geometrically separated with the two parameters $\Delta\Omega$ and $\Delta\alpha$. Then, these parameters, along with the inclination, have been optimized to find a configuration that exhibits a better worst GDOP. Thus, we ended with the following optimal parameters:

i = 53.19 degrees (originally 55 degrees)
$\Delta\Omega$ = 12.53 degrees (originally 60 degrees)
$\Delta\alpha$ = 42.36 degrees (originally 15 degrees)

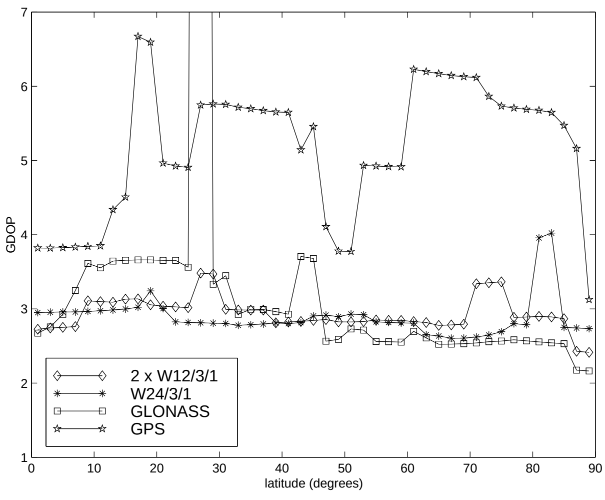

Figure 5: Comparison of the 24-satellite constellations

This configuration shows a very good worst GDOP of 3.48 (table 1 and figure 5), a better result than the other 24-satellite constellations. However, the continuous method of evaluation gives a surprising value of 4.04, far over the previous result. This last evaluation seems to indicate that the discretization is probably too crude in this case.

Hybrid LEO+GSO constellations

The second application is based on GNSS2, a European project intended to provide a civilian navigation service and driven mainly by ICAO requirements for non precision approach and the more stringent landing phases. One of the project's concept involves an hybrid constellation made of a big LEO constellation strengthened with a string of GSO satellites.

In this case, a global coverage is not required, and we will achieve only a +/- 68 degrees of latitude coverage (with the same precision than in the previous example). All the candidates constellations will be combined with 12 equidistant GSO satellites that circle the Earth. As a comparison base, we selected the constellation proposed by Alcatel[10], a Walker 70/7/6 with a semimajor axis of 7778 km and an inclination of 62.75 degrees).

Then, we computed the performances of all Walker constellations from 60 to 80 satellites (in combination with the GSO satellites of course). The table 2 summarizes the performances of the most interesting configurations (the constellations over 70 satellites do not offer a big improvement). Some of them have already been identified by Alcatel, but not the Walker 70/14/1 even though it is very effective. The performance index used by Alcatel is different and based on statistical target HDOPs and VDOPs, but we assume that the 70/14/1 should also perform well with this criterion. Moreover, the Walker 70/7/6 can provide very bad GDOP at the lowest latitudes (especially around 9 degrees, see figure 7). Therefore, a 70/14/1 looks to be a very good candidate.

Table 2 : Performances of LEO+GSO constellations

constellation	GDOP		
	mean	worst (1)	worst (2)
W60/12/2	2.16	23.4	>1000
W63/9/3	2.10	>10.8	661
W65/13/1	2.12	>6.68	13.4
W70/7/6	2.06	517	>1000
W70/14/1	2.05	5.83	14.2

But, worst GDOP values computed with the discrete algorithm should be taken carefully in this example, because of the very long period of repetitivity (LEO and GSO components do not have a similar orbital period). The continuous evaluation is here very useful and offers a quicker and a much more precise method to find the worst GDOPs, with the risk to focus on very small and non representative cases. But even there, a Walker 70/14/1 is still impressive.

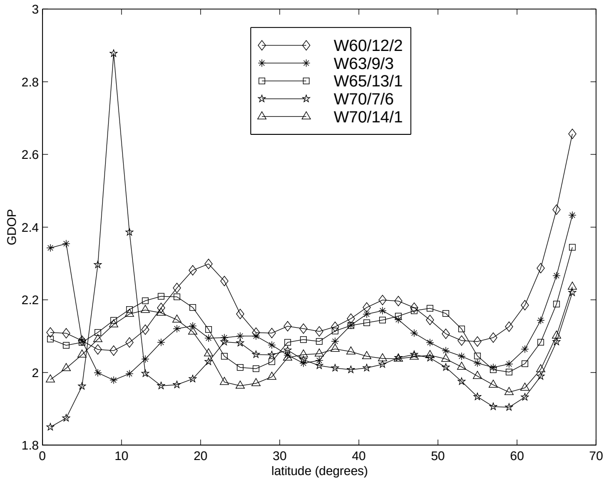

Figure 6: Mean GDOPs of the best hybrid LEO+GSO

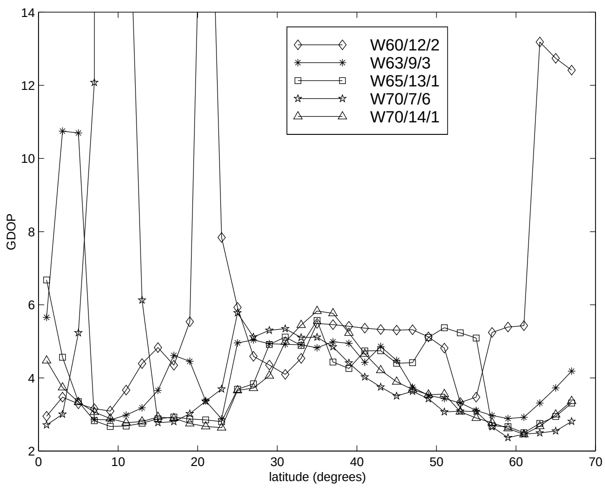

Figure 7 : Worst GDOPs of the best hybrid LEO+GSO

Conclusion

In this paper, we presented a complete methodology to identify and optimize a good constellation configuration for a navigation mission. The performance criterion is based on a weighted summation of the DOPs, and we can drive the optimization process according to both the mean or the worst DOPs for a precise tuning of the constellation. Moreover, we introduced a flexible type of constellation, the multi-Walker constellation, that is a combination of Walker constellations. The examples presented show that interesting results and improvements can be expected with this method.

For still better results, further developments should encompass the pseudovelocities (they can be very useful to improve the navigation precision with LEO constellations), performance degradations following satellite failures or additional cost factors like the number of launches or the number of spares.

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