# OPTIMAL MANEUVERS FOR STATION KEEPING FOR A GIVEN CONFIGURATION OF THE SATELLITE CONSTELLATION


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## Abstract

A solution technique of constellation configuration maintenance is presented. The configuration of a constellation with many satellites is characterized by their mutual deviations of orbital elements. These deviations should satisfy to given constraints on a long time interval (from several days to two years). An optimization of the solution is reached by choosing of satellite positions at the end of this time interval. We minimize the summary characteristic velocity as well as the objective function enabled to provide uniform consumption of the fuel for all satellites. To find the optimal solution total number of used impulses is also minimized if it does not increase appreciably the objective function magnitude. The needed accuracy is provided by an optimization of maneuver parameters using the iteration procedure based on the high-efficient THEONA software. To develop this technique we have used our experience with ballistic-navigational control of real spacecrafts for many years.



***Key words****:* Satellite Constellation, Configuration Maintenance, Maneuver Optimization, CONSTELL Software, THEONA Software.


## Introduction

The problem of constellation configuration maintenance is considered. The satellites move on near-circular orbits with closes radii. Their mutual deviations on inclination $\boldsymbol{i}$, right ascension of ascending node $\boldsymbol{\Omega}$ and latitude argument $\boldsymbol{u}$ should not surpass the given values (depended on the station keeping constraints)[1].

We offer the technique of problem solution which enables to co-ordinate a movement of the system on a given time interval (from several days to two years). A great number of satellites and a long interval of consideration gives some difficulties for the problem. This peculiarity produces high requirements to efficiency of used methods (for optimization and orbit prediction) as concerns their rapidity and accuracy. Good convergence of the iteration procedure (for optimization solution) is provided also by the choice of controlled parameters of the satellite constellation.

## Description of the optimization technique

It is assumed that the time-dependence of mutual deviations ($\boldsymbol{i}$, $\boldsymbol{\Omega}$, $\boldsymbol{u}$) is near-linear function. In this case, if the controlled parameters of each satellite are deviated from corresponding ones of the base orbit not more than 1/2 of precision values of system state keeping at the beginning and the end of the maintenance interval, they satisfy to admissible values within interval. Such approach allows to determine the impulses parameters for each satellite independently of the others, so it symplifies essentially the task.

The developed methods of maintenance optimization are realized by the CONSTELL software. Its organigram is shown in Fig.1. To present our technique of above-mentioned problem it is convenient to use the description of subroutines of this software.

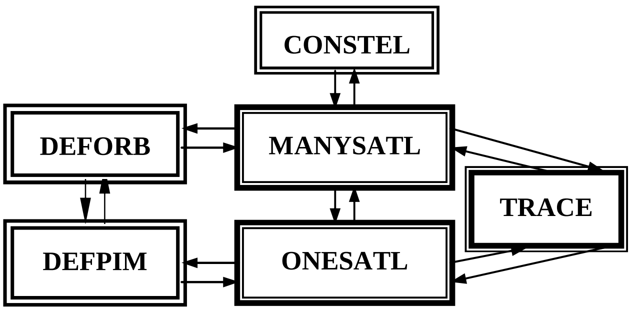


**Figure 1: Organigram of the CONSTELL software**

The CONSTEL subroutine serves to provide an interface between input-output information and arrays of the MANYSATL subroutine for calculations.

At the beginning of the MANYSATL subroutine, the initial conditions (initial orbital elements) of all satellites are propagated till to the start moment of the maintenance interval. To integrate the equations of motion it is always used the TRACE subroutine based the THEONA software (see below). After, it is found such initial state of the base orbit as maximal deviations (for every controlled parameters) would have equal magnitude with different signs.

Initial conditions of all satellites are propagated till to the end of maintenance interval and it is determined the values of controlled parameters. The parameters of each satellite correspond to the point in the (***i***, ***Ω***, ***u***) space. An example for the system of 6 satellites is presented in Fig.2, where a satellite number is posed about corresponding point.

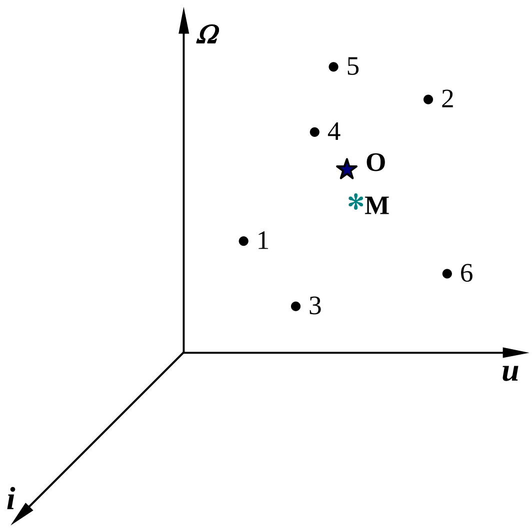


**Figure 2: Example of the system parameters**

The DEFORB subroutine finds optimal values of the base orbit parameters. As a rule, the optimal point **O** (see Fig.2) is situated near from the point **M** corresponded to the mean of the values of controlled parameters. The optimal values are considered as the values of the base orbit parameters for which the functional $F = \sum_{j=1}^{n} \frac{\Delta V_j}{\Delta V_j^*}$ is minimal, where $n$ is the number of satellites, $\Delta V_j^*$ is disposed impulse (the summary characteristic velocity) of $j$-th satellite, $\Delta V_j$ the impulse needed for its transfer into the base orbit. A minimization of such functional permits to evenly distribute the fuel expenditure between satellites.

The parameters of impulse are determined in the DEFPIM subroutine. If it is necessary to change only one of the orbit elements, the value of corresponding component of the impulse is calculated by the formula:

$$\Delta V_j = -\frac{\delta Q_j}{dQ_j / d\Delta V_j} \qquad \textbf{(1)}$$

where $\delta Q_j$ is the deviation of the corrected parameter of the real orbit of $j$-th satellite from needed value of corresponding parameter of the base orbit, and the derivation $d\, Q_j \,/\, d\, \Delta V_j$ demonstrates an influence of the corresponding impulse component to the corrected parameters at the end of maintenance interval. This derivation is also calculated in the TRACE subroutine.

If we correct only the inclination, the impulse is applied on equator; to correct the right ascension of ascending node ***Ω*** the application point of impulse is a pole of the orbit; to correct a location along orbit the impulse is applied on one of apsidal points for decreasing of orbit eccentricity. If we correct simultaneously the deviations of ***Ω*** and latitude argument ***u***, then the impulse is applied on the orbit pole, its transversal and lateral components are determined from the system of 2 linear equations with 2 variables. In other cases, additionally, the transversal component $\Delta V_{Tj}$ of the impulse is determined from formula (1), its application angle $\boldsymbol{u_j}$ and lateral component $\Delta V_{Wj}$ are found from the system:

$$\delta i_j = \frac{r_j}{\sqrt{\mu p}} \cdot \cos u_j \cdot \Delta V_{Wj} \qquad \textbf{(2)}$$

$$\delta \Omega_j = \frac{r_j}{\sqrt{\mu p}} \cdot \left( \frac{\sin u_j}{\sin i} + \eta u \cdot \sin i \cdot \cos u_j \right) \cdot \Delta V_{Wj}$$

$$-4 \frac{r_j}{\sqrt{\mu p}} \cdot \eta u \cdot \Delta V_{Tj} \qquad \textbf{(3)}$$

where $\mu$ is the Earth gravity constant, $p$ the orbit parameter, $r_j$ the corresponding distance from the center of the Earth to the satellite, and the coefficient $\eta = -\frac{3}{2} J_2 \left( \frac{R_e}{p} \right)^2$, $J_2$ – second zonal harmonic, $R_e$ the equatorial radius of the Earth.

So we can correct all 3 deviations by one impulse and there is not the situation when the correction of one parameter (e.g. ***i***) increase the deviation of other parameter (e.g. ***Ω***).

Moreover, it is considered the cases when the impulse magnitude does not satisfy to admissible limitations (bigger than maximum or smaller than minimum). The choice of the base orbit parameters (by the DEFORB subroutine) takes into account the limitation on the number of spacecrafts which have the possibility of manoeuvring in given time interval. The parameters of the base orbit could be set in the input information. Then the satellite system motion will be co-ordinated with the movement of this given orbit.

After choosing of optimal parameters of the base orbit by the iteration procedure in the ONESATL subroutine, it is realized a determination of the impulse parameters with which chosen base orbit will be formed with needed accuracy. In this stage each satellite is considered separately.

The ONESATL subroutine uses the target orbit that coincides with the base orbit in first iteration. After determination (by the DEFPIM subroutine) of parameters of the impulses that form the target orbit, the TRACE subroutine predicts precisely the parameters of actually formed orbit. The deviations of these parameters from the base orbit parameters are calculated and compared with admissible ones. If they are bigger than admissible values, the parameters of the target orbit are changed by calculated corrections. Then it is found new values of impulse parameters, and so on, until the deviations from the base orbit become smaller than admissible values.

This procedure is repeated for each satellite until the whole system transfers to chosen base orbit.

High accuracy of the maintenance of state of the satellite system in longtime interval is provided by the use of the Numeric-Analytical integration (the TRACE subroutine) in the iteration procedure.

## Numeric-Analytical integration

The TRACE subroutine uses the high-efficient THEONA software based on the Numeric-Analytical Satellite Theory[2,3]. It predicts the satellite orbital elements (e.g. osculating ones) and their derivatives on several parameters to the needed moment of time or latitude argument. The integration of the satellite motion equations is realized semi-analytically and takes into account all essential perturbations: the Earth's gravity field (with arbitrary number of harmonics), the air drag (with full standard models of atmospheric density), the gravitational influence of the Moon and the Sun, the solar light pressure with the shadow effect. These forces are necessaries (and sufficients) for the orbits of satellite constellation with altitudes between 1400 and 1500 km (e.g. SKYBRIDGE, TELEDESIC etc.).

Good accuracy of the calculations based on the THEONA software accompanies their high rapidity. This is arrived by using the special function technology (such as the Jacobi functions, modified Legendre functions, and additionally, new special functions that are a generalization of well-known Hansen coefficients) based on differential-recurrent relations.

High rapidity of the TRACE subroutine integration permits to obtain the problem solution for little time (even if we work with longtime intervals of the state maintenance of the system consisted of several tens of satellites).

How the errors (due to the orbit parameters knowledge and the impulse realization) influences on the system state, this question may be studied by the PROGDEV subroutine. This program allows to compute with given step of time the mutual deviations of the satellite orbital parameters taking into account all above-mentioned perturbations.

Now it is realized the work in order to enrich the CONSTELL software by the analytical algorithmes used in Keldysh Institute of Applied Mathematics for ballistic support of the program of piloted spaceflights to solve the rendez-vous problem[4]. This will allow to use this software for calculating of manoeuvres of the satellite constellation forming and the satellite transfer from one point of the constellation to other point (the substitution).

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