COMBINED CONTROL OF THE OPERATIONAL MARS EXPRESS ORBIT AND THE SPACECRAFT ANGULAR MOMENTUM

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ABSTRACT

The orbit keeping of Mars Express during the observation phase uses the delta V produced by the momentum off-loading manoeuvres to try to get as close as possible to a reference orbit, called "frozen orbit", which implements all the orbit requirements of the mission and at the same time maximizes the surface coverage. This reference orbit is propagated considering perturbations and includes small, optimized manoeuvres with limited size at its apocentres to tune the orbit period.

For this orbit keeping the spacecraft is oriented in an attitude such that the resulting delta V of the off-loading manoeuvres is parallel to the velocity (either along or against), maximizing in this way the orbit period change. The objective is to fly over the sub satellite point of the frozen orbit pericentres.

This paper presents the design of the frozen orbit, the method used to keep the actual orbit as close as possible to the reference and the operational results achieved.

1. INTRODUCTION

Mars Express was launched on June 2nd, 2003 to a direct transfer orbit to Mars, where it arrived on December 25, after releasing the lander, Beagle 2, six days before. The Mars Orbit Insertion (MOI) manoeuvre was followed by a series of manoeuvres to bring the satellite to the observation orbit, where it arrived on January 28th, 2004.

In the observation orbit the spacecraft must fulfil the orbit requirements derived from the scientific instruments and at the same time maximize the scientific return of the mission. These orbit requirements are fulfilled indirectly, by trying to fly as close as possible to a reference orbit, which fulfils itself the requirements and maximizes the observed surface. In order to do that, the velocity increment produced by the momentum wheel off-loading manoeuvres (WoL) is used. These frequent manoeuvres are unbalanced thrusters actuations producing a residual delta V. These manoeuvres are executed around apocentre to minimize the interference with the scientific observations, which are performed mainly around pericentre where the instruments resolution is the highest. In this paper the operational implementation of that control is explained. We start by describing how the reference orbit is constructed. It follows the description of the operational implementation of the orbit control with WoL's and after that the results achieved are presented.

2. THE REFERENCE OR "FROZEN" ORBIT

2.1 Operational orbit selection

The operational orbit of Mars Express was selected to maximize the scientific return of the mission and at the same time satisfy the requirements of the instruments. The requirements and preferences of the instruments can be summarised as follows:

- Coverage of the whole Martian surface from low altitude
- Different ground track spacing for the different instruments
- Preference for different illumination conditions (sun elevation at the sub-satellite point) for different instruments. For example, the High Resolution Stereo Camera (HRSC) and OMEGA prefer day side viewing at sun elevations between 30 and 60 degrees, whereas the radar prefers night side viewing
- Low altitude coverage of both Martian poles with good illumination, but also during night
- Deep night coverage at low altitude in the equatorial region (radar)
- A relay service to the Beagle 2 lander from the beginning of the mission. At least one contact per sol would be desirable, as well as a specific data volume per week
- The scheduled radar antennae deployment operations must be taken into account

The main mission constraint is the available launch mass and thus the available delta V. With this limiting

factor, the solution was a high eccentricity, almost polar orbit called G3-ub-eq100, which includes an orbital period change. The high inclination (initial value, 86.6 degrees, see Table 1) combined with the pericentre regression guarantees coverage of the whole Martian surface from low altitudes.

The orbit G3-ub-eq100 is made up of two parts. Both are almost resonant to fulfil the ground track spacing requirements. The first one (G3-u) is near the 13/4 resonance (13 orbit revolutions every 4 Mars rotations or "sol") and the initial pericentre is almost over the equator. The initial orbit elements are shown in Table 1. The second part is near the 11/3 resonance, and therefore it has a lower period. The period change is achieved by two apocentre lowering manoeuvres, executed about 100 days after the start of the frozen orbit. This 100-day period was selected taking into account the operations for the radar (MARSIS) antennae deployment before the pericentre passes over the South Pole. Observability requirements with the lander, Beagle-2, to allow for communications (nominal landing site, latitude 11.6 deg north, longitude 90.74 deg east) are also taken into account in the selection of the initial phase.

The initial argument of pericenter and right ascension of the ascending node (RAAN) were selected such that the orbit can be reached with the available delta V and at the same time provide a good balance between day and night viewing for the whole mission.

Epoch (MJD2000, TDB)	1473.66355847		
Pericentre Radius (km)	3669.860		
Apocentre Radius (km)	15039.293		
Inclination (deg))	86.583		
RAAN (deg)	228.774		
Argument of Pericentre (deg)	-2.019		
True Anomaly	0.001		

 Table 1. Frozen orbit initial osculating elements (Mars

 Mean Equator, IAU of date system)

2.2 Frozen orbit propagation

The frozen orbit takes into account all the requirements provided by Mission Analysis in terms of initial parameters and ground track repeat cycle, and at the same time the observed surface is maximized. It has been integrated considering all relevant perturbations acting on the spacecraft, together with small manoeuvres at each apocentre, either against of towards the velocity, to modify the orbital period and fulfil the minimum overlap constraint. This constraint, which determines the ground track separation, derives directly from the High Resolution Stereo Camera requirements. It specifies a given overlap (5%) between consecutive swaths during the high resolution surface mapping, which is assumed to be mostly done below 800 km altitude in nadir pointing spacecraft mode. The HRSC field of view is 11.9 degrees.

The selection of the sizes of the apocentre manoeuvres is a constrained optimization problem in which the overlap requirement is ensured by reducing the HRSC field of view from 11.9 to 10.7 degrees (10 % reduction to 5.35 degrees semiangle) and imposing the no-gap constraint between every pair of two consecutive swaths (this constraint applies to the point of minimum overlap, which must be found for every pair of consecutive swaths). The cost function to maximize is the covered surface in terms of longitude.

The manoeuvre size at every apocentre is limited to a change of 20 meters of semimajor axis per revolution. Occasionally this value is increased to 30 or 40 meters near gravity anomalies. The equivalent size of these manoeuvres is shown in Table 2. These manoeuvre limits are of the order of magnitude of the hypothetical momentum wheel off-loading manoeuvre that would be needed to off-load the momentum accumulated in the wheels in every revolution (the WoL manoeuvres are performed every several revolutions). The manoeuvre size limits produce that in some revolutions the overlap is larger than strictly needed, and therefore the cost function is penalised.

Table 2. Maximum sizes of apocentre manoeuvres

Orbit	G3-u	G3-b		
Semimajor axis change (m)	20	20	30	40
Size (mm/s)	4.66	5.00	7.50	9.95

The manoeuvres at apocentre are only applied when the pericentre is between -60 and 60 degrees latitude. Outside this region the trajectory is a free drift propagation. The reason for this is that closer to the poles the swaths are much wider in terms of longitude and the optimization would require much larger manoeuvre sizes to change the period before the no-gap constraint is active. Furthermore, for the same reason, the polar regions will be well covered and the optimization would not bring any additional gain.

The frozen orbit is propagated well before the corresponding actual operations are prepared (typically six moths).

Due to the limitations of the communications link to Earth, a global surface coverage at the resolution of the instruments from these low altitudes cannot be obtained within the one or two Martian years planned for the Mars Express mission. In any case, the frozen orbit concept explained here is applied always because this orbit control is needed to observe large coherent areas within a not too long time interval, e.g., without too long seasonal variations, and with similar illumination conditions.

3. OPERATIONAL ORBIT CONTROL

In the prediction of the operational orbit the manoeuvre optimization sub-system provides an orbit for the following days (as long as required, typically 15 days) that includes the WoL's planned for that period. These manoeuvres are executed in allocated time intervals for orbit maintenance. These are time windows with a duration of one and a half hours, selected by the mission planners (in charge of scheduling the scientific observations), in such a way that the interference with other operations is minimized. In this time the S/C performs the attitude manoeuvre to put its Z-axis parallel to the velocity, actuates the thrusters to off-load the momentum wheels and recovers its initial attitude with another slew manoeuvre. The orbit maintenance windows are centred at apocentre of the frozen orbit and are frequent enough as to avoid wheel saturations and the corresponding uncontrolled autonomous WoL.

The selection of the WoL manoeuvres direction included in the orbit prediction (direction either along or against the velocity) is done in such a way that it tries to bring the actual orbit as close as possible to the frozen orbit, in the sense that the S/C flies over the sub-satellite points corresponding to the pericentre of the frozen orbit. This selection is a discrete optimization problem whose cost function is the sum of the phase error between the frozen and actual orbits corresponding to every pericentre passage included in the orbit prediction (see Eqn. 1). The phase error definition depends on the pericentre latitude:

- 1. For latitudes between -70 and +70 degrees the phase error is defined as the difference in longitude between both ground tracks measured at the latitude of the frozen orbit pericentre.
- For latitudes outside that region the phase error is measured as the difference in argument of latitude (true anomaly + argument of pericentre) between both orbits at the frozen orbit pericentre passage time

The contribution to the cost function of every phase error is taken in absolute value. The maximum phase error is considered with and additional weight as shown in Eqn. 1.

$$F = \sum_{i=1}^{n} \left| \Delta \lambda^{i} \right| + W_{\max} Max \left\{ \Delta \lambda^{i} \right\}$$
(1)

In Eqn. 1. $\Delta\lambda$ is the phase error, *n* is the number of revolutions and W_{max} is the weight applied to the maximum phase error to consider an additional cost for the function.

The variables in the optimization problem are the WoL manoeuvres, which can take only two discrete values, along or against the velocity, the size cannot be chosen. An approximate value of the WoL sizes is needed (a prediction of each one is the best case, otherwise, a statistical value of the last executed WoL's can be taken, and all are considered with the same size).

The optimization problem requires to evaluate the cost function for every combination of WoL directions. This means the propagation of the complete trajectory 2^n times, with n the number of WoL manoeuvres (a typical value for n is 15). In order to reduce the computational time the optimization is performed in two steps:

- 1. In the first step a continuous optimization problem is solved with a gradient method. The manoeuvres are allowed to have any intermediate value between the corresponding positive (along the velocity) or negative (against the velocity) discrete values. The resulting orbit is usually very close to the reference one, because overshoots are avoided. This solution is used as a first guess for the next step.
- 2. In the second step a modified simple grid method is applied. Every combination of WoL directions is tried, and the cost function shown in Eqn.1. is replaced with its first order approximation for a faster evaluation of the cost function. Every phase error is linearised as shown in Eqn. 2.

$$\Delta \lambda^{i} = \Delta \lambda_{0}^{i} + \sum_{j=i}^{m} \frac{\partial \lambda^{i}}{\partial \Delta V^{j}} \left(\Delta V^{j} - \Delta V_{0}^{j} \right) \quad (2)$$

In Eqn. 2. $\Delta \lambda_0^{i}$ and ΔV_0^{j} are the phase errors and manoeuvre sizes, respectively, resulting from step 1. *m* is the number of WoL manoeuvres, and each ΔV^{j} will have the maximum or minimum value (negative) depending on the combination of WoL directions for which the cost function is evaluated.

The optimization of the WoL directions is done twice per week. When the optimization is finished, the WoL's corresponding to the next 3 to 4 days are prepared by the command generation sub-system. This corresponds to about 3 or 4 WoL's. The remaining WoL directions are not used as they will be updated in the next optimization.

4. OPERATIONAL RESULTS

4.1 Nominal operations

The operational method described in previous paragraphs has been applied since the arrival of the satellite to the operational orbit G3-u on January 28th, 2004. As an example, Fig 1 shows the results of this strategy for the period from the 14^{th} of February to the 10^{th} of March 2004. The solid lines show the phase difference (longitude) related to the frozen orbit for each pericentre (left scale). The dashed lines show the delta V size of the WoL manoeuvres (right scale). The crosses (x) show the manoeuvres sizes at each pericentre of the frozen orbit (right scale).



Fig. 1. Orbit control at the beginning of the observation phase

It can be seen in Fig. 1 that the manoeuvres corresponding to the frozen orbit are positive and negative. The WoL's are alternating positive and negative as well, meaning that they are large enough to control the phase, and in case of overshooting the following manoeuvre or manoeuvres are used to correct the drift. For the selection of the WoL directions the future WoL sizes were considered constant, based on an averaged value of the previous ones.

It remains to be seen that following this strategy the overlap constraint is fulfilled. This can be seen in Fig. 2, which shows the minimum overlap around pericentre between each swath and the previous consecutive, produced 13 revolutions before. The overlap is always positive, meaning that the overlap constraint is fulfilled.



Fig. 2. Swath overlap at the beginning of the observation phase

Fig. 3 shows the accuracy achieved in the orbit control for the second part of the observation phase, orbit G3-b. The maximum phase error, around 0.04 degrees, is similar to the one shown in Fig 1. It can be seen as well that the frozen orbit does not contain any manoeuvre before August, 3^{rd} . This is due to the fact that before that time the pericentre is bellow -60 degrees latitude, and it is then when it crosses that pararell towards the equator. During the days shown in Fig. 3 most of the manoeuvres of the frozen orbit are negative because the swath width in terms of longitude reduces rapidly and a faster period change is needed.

The minimum swath overlap corresponding to Fig. 3 is shown in Fig. 4. The overlap starts with a very high value, and reduces as the pericentre moves over the mentioned parallel, taking after that a more stable value. This is the effect of the change of the frozen orbit propagation occurring on August 3rd.



Fig. 3. Orbit control for orbit G3-b



Fig. 4. Swath overlap for orbit G3-b

4.2 Non nominal operations

The minimum overlap constraint cannot be guaranteed if the WoL's cannot be executed as described previously, or in case that an unexpected delta V is applied, as in case of a safe mode. Fig. 5 shows the effect of a safe mode, which occurred on March 11th, in which a large delta V was applied, aggravated by the fact that for some days the WoL's could not be executed in the optimized direction. The corresponding minimum overlap is shown in Fig. 6. The safe mode caused an irregular distribution of the swaths, with gaps during the first days after the safe mode and a too large overlap the following days.



Fig. 5. Effects of a safe mode on the orbit control



Fig. 6. Minimum overlap after the safe mode

5. CONCLUSIONS

The frozen orbit is a powerful concept for the operational orbit keeping of Mars Express. It has the following advantages:

- It simplifies the manoeuvre optimization process, because just by following the reference orbit the overlap requirement is fulfilled during normal operations
- It maximizes the observed surface
- It allows a long term planning of the operations
- As the frozen orbit includes all the relevant perturbations, it is a good long term prediction of the evolution of relevant parameters, like argument of pericentre or inclination
- It simplified the target for the manoeuvre optimization operations related to the Mars Orbit insertion and following manoeuvres

The operational results presented here demonstrate that the frozen orbit concept guarantees the fulfilment of the requirements, and that this orbit can be flown with the delta V produced by the WoL's, without additional manoeuvres, therefore saving fuel and prolonging the mission lifetime.

6. **REFERENCES**

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