FORMATION FLYING TRANSITION BEHAVIOUR TO LOCALIZED AND DECENTRALIZED CONTROL LAWS

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ABSTRACT

The paper presents how the formation is controlled to the intended shape by a decentralized control. The formation behaviour is dealt via a z-transformation method and a uniformly convergent strategy that each spacecraft performs is proposed. Not only one-dimensional but tow-dimensional examples are shown. Since the strategy is highly flexible, it is applicable to a variety of the formation flying space missions.

1. INTRODUCTION

Formation Flying has become very common recently and its keeping strategies are now of great interest in astrodynamics these days. It may be simply true that the optimal station keeping is realized only when every spacecraft information is shared by every spacecraft in the formation. This is the case called 'fully informed' state and the strategy taken results in a centralized control. However, in case the number of spacecraft constituting a formation becomes infinitely large, since the communication load becomes divergent and also since the states uncertainty may deform the relative geometry before the entire information is exchanged and shared amid the formation, above mentioned 'fully informed' state will not appear actually.

Instead of this cumbersome 'fully informed' control law, it is practical to apply a localized control law that requests the relative motion among a few spacecraft nearest to and adjacent to each other. In this case, the strategy will be a decentralized one. An important phenomenon observed here is a non-optimal transition behaviour, via which the formation is deformed and settles to the targeted formation shape, the goal. This 'Partially informed' control schemes sometimes make the transition behaviour not uniformly converged to the goal and the settle time increases. Besides, the time delay via which the relative motion spreads to the entire formation significantly affects to both the settle time and transition behaviour. This feature also leads to the traffic jam phenomenon in highways on the ground.

This paper presents how the formation control structure is expressed explicitly and how a kind of traffic jam appears analytically.

2. Z-TRANSFORMATION APPLIED TO SPATIAL INFORMATION EXPRESSION

A Z-transformation is a classical analysis strategy for discrete time systems to represent a series of impulses in time-domain. The same method is in this paper revisited and used to express the formation flying information that is inevitably defined at discrete nodes.

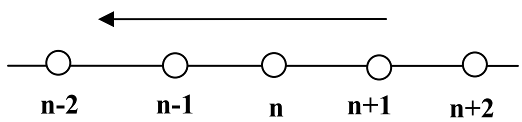

Fig. 1 1-dimensional Formation (Car Train)

Fig. 1 above shows a typical one-dimensional formation such as a car train running on a highway. The most fundamental expression rule in z-transformation is in impulses. The z-transformation of a single impulse is unit, 1. And here, shifting an impulse backward is done by an operator of 1/z, while forward shifting is by z. Therefore, when a series of constant impulses from the top position at zero is given by

$$y^*\left(1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}\cdots\right)=y^*\frac{1}{1-\frac{1}{z}}=y^*\frac{z}{z-1}, \qquad (1)$$

where y* denotes the intensity of the impulses.

3. ONE-DIMENSION FORMATION – CARS INTERVALS

The most fundamental equations of motion are those for the distance intervals between the cars running on a highway. Here denote y_n be the interval between the (n-

1)-th car and the n-th car and also denote u_n be the velocity increment applied to n-th car as a control input, the equations of motion are written by

$$\dot{y}_1 = -u_1,\ \ \dot{y}_2 = -u_2 + u_1,\ \ \dot{y}_3 = -u_3 + u_2, \cdots . \quad (2)$$

Introduction of z-transformation makes this written simply by

$$\dot{y}_z = \left(\frac{1}{z} - 1\right) u_z . \quad (3)$$

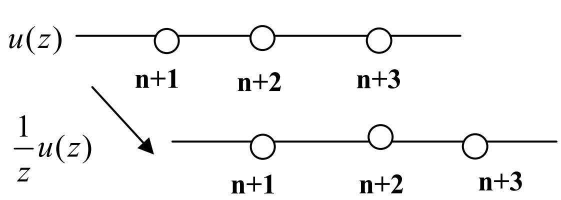

Fig. 2 Shifting Train backward

As Fig. 2 shows, it should be noted that the acceleration applied to the leading car appears as being shifted backward via 1/z operation. The most straightforward control strategy may be found as

$$u_n = k\left(y_n - y^*\right). \quad (4)$$

This is what usual drivers do and is to make the distance to the leading car be controlled to a certain appropriate distance y*. Similarly at this step, instead of acceleration input to each car, the equations of motions including feedback control are expressed by those rewritten via the z-transformation. Using the intervals stream y_z, they are

$$\dot{y}_z = \left(\frac{1}{z} - 1\right) u_z,\ \ u_z = k\left(y_z - \frac{z}{z-1} y^*\right). \quad (5)$$

Note even what this expresses is a kind of decentralized control where no one collects every information of the formation, while entire formation constituted by the participants (cars) perform the each task implicitly supposed to do trying to run smoothly. This is one of the most basic decentralized formation control examples already established. The closed loop response in this example is obtained by

$$\dot{y}_z = k\left(\frac{1}{z} - 1\right)\left(y_z - \frac{z}{z-1} y^*\right). \quad (6)$$

Obviously, the intended formation appears if $\dot{y}_z = 0$, when the steady state solution is an infinite series of constant impulses as eq. (6) indicates. The solution of this is easily obtained in time domain as

$$y_z = \exp\left(kt\frac{1-z}{z}\right) y_{z0} + \frac{z}{z-1} y^* \left(1 - \exp\left(kt\frac{1-z}{z}\right)\right), \quad (7)$$

where y_{z0} denotes the initial interval stream which may be expressed as follows, in case they are constant distance interval stream y_0:

$$y_{z0} = \frac{z}{z-1} y_0 \quad (8)$$

3.1 Traffic Jam Phenomenon

First of all, the case when y* is zero is studied. This case correspond to the situation when the top car stops or decelerates and the traffic jam builds up. Intrinsically, in view of the first term in eq. (7), the behaviour of

$$y_0 \frac{z}{z-1} \exp\left(kt\frac{1-z}{z}\right) \quad (9)$$

seems to describe the monotonous decay in terms of time. However, it is not true. What happens is observed through the inversed z- transformation. The above equation is decomposed and rewritten as:

$$\begin{aligned} y_0 \frac{z}{z-1} \exp\left(kt\frac{1-z}{z}\right) &= y_0 e^{-kt} \frac{z}{z-1} \exp\left(\frac{kt}{z}\right) \\ &= y_0 e^{-kt}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots\right)\left(1 + \frac{1}{z}(kt) + \frac{1}{z^2}\frac{(kt)^2}{2!} + \frac{1}{z^3}\frac{(kt)^3}{3!} \cdots\right) \\ &= y_0 e^{-kt}\left\{1 + \frac{1}{z}(1+kt) + \frac{1}{z^2}\left(1 + kt + \frac{(kt)^2}{2!}\right) + \frac{1}{z^3}\left(1 + kt + \frac{(kt)^2}{2!} + \frac{(kt)^3}{3!}\right) + \cdots\right\} \end{aligned} \quad (10)$$

Since the $1/z^m$ indicates the m-times shifting backward, rear cars retains the initial formation shape information more. In other words, this is interpreted as the propagation takes step by step from front to rear direction and this transition results in.

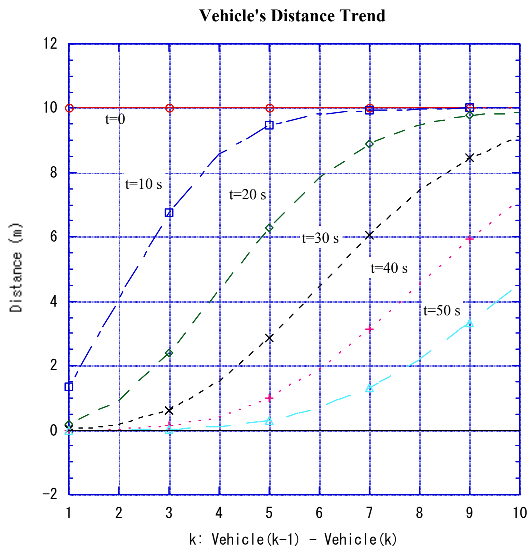

Fig. 3 Car Train Response to Jam

Fig. 3 above presents a numerical simulation in case the target interval is taken zero. This shows how traffic jam grows and the intervals become short gradually. An important point is in the fact that the propagation takes 1/k second, which is imbedded in the servo dynamics here. Later discussion provides how the delay time effect that contributes to this phenomenon. The results in Fig. 3 is well reviewed by eq. (10) whose characteristics are depicted in Fig. 4 below.

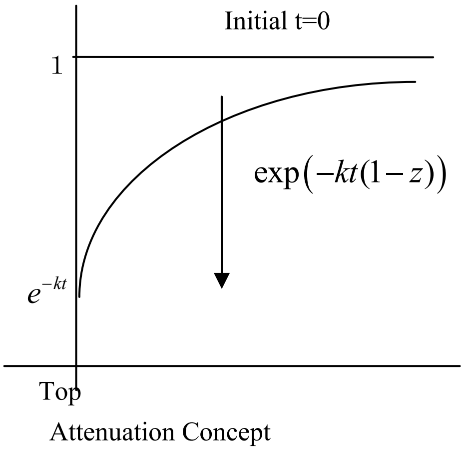

Fig. 4 Qualitative Interpretation

3.2 Resume of Traffic Jam

If the behaviour is evaluated from the moment when every initial interval is zero and the top car starts constant velocity cruise. This corresponds to the case when the traffic jams starts resume. The transition is described by the second term in eq. (7), where the interval stream y_z converges according to the characteristics of $1-\exp\left(kt\frac{1-z}{z}\right)$ operator.

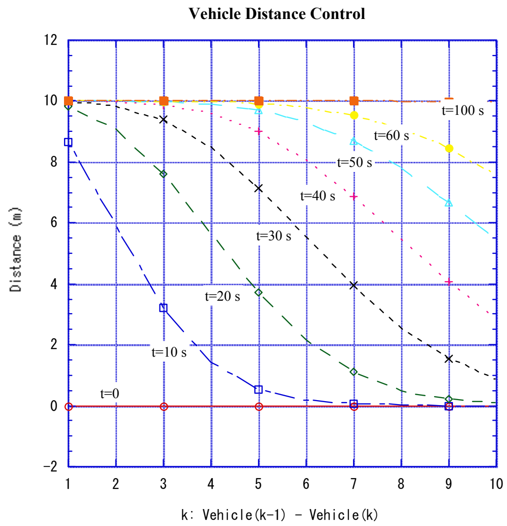

Fig. 5 Car Train Response to Resume

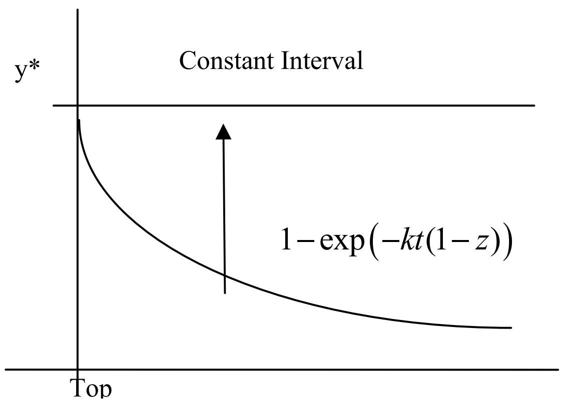

Fig. 6 Qualitative Response Resume

Above figure shows the results obtained by the numerical simulation. The transition occurs in reversed way to the case the traffic jams starts. An intuitive and qualitative understanding is glimpsed by Fig. 6 below that illustrates the tendency that appears in eq. (10).

There might be questioned why intervals propagate and take some time, while each car tries to maintain a constant interval. This apparently seems strange, however, an interpretation looms when the control effort applied is consequently written by

$$\dot{y}_n = -u_n + u_{n-1} = -k\left(y_n - y_{n-1}\right). \tag{11}$$

Here no targeted interval y* appears explicitly in the dynamics. Actually, as the intervals between cars become constant even though they are slightly different from those specified, the formation is not updated quickly as eq. (11) shows. Only the information from leading cars propagates and updates the shape.

4. DECENTRALAIZED UNIFORM TRANSITION STRATEGY

Generally speaking, the formation transition example here is governed by the following equation.

$$\dot{y}_z = \left(\frac{1}{z} - 1\right) u_z(y_z, y^*) \tag{12}$$

How to design u_z determines the transition behaviour. The primary reason why the transition occurs no-uniformly is in the equivalent gain contains spatial information z. Consequently, if the gain is tactically chosen, a uniform transition may be expected. If the following strategy is introduced,

$$u_z = \frac{z}{z-1} k \left(y_z - \frac{z}{z-1} y^* \right), \tag{13}$$

the formation response is written as:

$$\dot{y}_z = -k\left(y_z - \frac{z}{z-1} y^* \right) \tag{14}$$

Since the closed loop gain portion excludes z variable, the solution is simply

$$y_z = \exp(-kt) y_{z0} + \frac{z}{z-1} y^* \left(1 - \exp(-kt)\right). \tag{15}$$

This transition proceeds uniformly and is ideal. Investigating the structure of this strategy is beneficial and rewarded. This strategy accumulates the acceleration input to the leading cars.

Since $(z-1)u_z = zk\left(y_z - \frac{z}{z-1} y^* \right)$ is equivalent to $(1-\frac{1}{z})u_z = k\left(y_z - \frac{z}{z-1} y^* \right)$,

$$(1-\frac{1}{z})u_z = u_n - u_{n-1} = k\left(y_z - \frac{z}{z-1} y^* \right). \tag{16}$$

This states the accumulation structure of

$$u_n = u_{n-1} + k\left(y_z - \frac{z}{z-1} y^* \right). \qquad (17)$$

It should be again noted 1/z operation shifts the stream backward.
Alternatively, this strategy is obtained from the equations of motion. In

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \bullet \end{pmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ & & & \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \triangleq A\mathbf{u} \triangleq \mathbf{v}, \quad (18)$$

what is intended is to have $\mathbf{v} = -k\left(\mathbf{y} - y^*\right)$ directly so that the stream can converge uniformly. It is the inversion of matrix A and since

$$A^{-1} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 \\ -1 & -1 & -1 & -1 \end{bmatrix}, \qquad (19)$$

the control strategy is expressed as

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_1 + y_2 - y^* \\ u_2 + y_3 - y^* \\ u_3 + y_4 - y^* \end{pmatrix} = k \begin{pmatrix} y_1 - y^* \\ y_1 + y_2 - 2y^* \\ y_1 + y_2 + y_3 - 3y^* \\ y_1 + y_2 + y_3 + y \ - 4y^* \end{pmatrix}. \quad (20)$$

Note this equation actually corresponds to the above mentioned method in eq. (17). The essence lies in obtaining the distance from the top car at each car position by relaying the foreside information. And this is performed also in decentralized manner.
The inversion of A matrix corresponds to the centralized control, while the decomposed and rewritten strategy in eq. (17) replaces it.

In a sense, this strategy requests the transmission capability equipped with at each car. However, the centralized control also requests the direct communication between the top car and each car. In terms of the communication resource point of view, the latter centralized way requests a larger resource which may need high power transmitter aboard. From this point of view, the former decentralized scheme is better, since each car has only to be equipped with smaller radio instruments. It might be concluded that the decentralized and relayed control will be a solution to the formation transition from its nature.

There is, however, a drawback found for the decentralized method. Any communication malfunction between the cars will result in the formation disruption at that point. But another view may state such a division may be better from fault tolerance point of view.

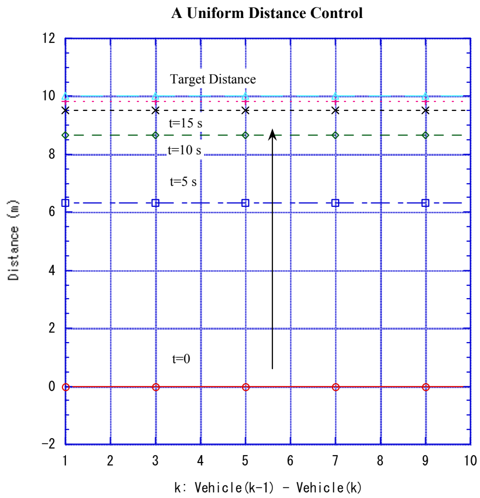

Fig. 7 Uniform Convergence to Target

Fig. 7 above provides a monotonous behaviour, which in a sense presents a uniform convergence of the stream.

4.1 Feedback of Interval to Rear Car

As eq. (11) points out, another strategy of using the interval information to the rear car apparently may help the transition. However, this is not true. The control law of

$$u_n = k\left((y_n - y^*) - (y_{n+1} - y^*)\right) = k\left(y_n - y_{n+1}\right), \quad (21)$$

which is

$$u_z = k(1-z)\left(y_z - \frac{z}{z-1} y^* \right), \qquad (22)$$

makes the response as

$$\dot{y}_z = k\frac{(1-z)^2}{z}\left(y_z - \frac{z}{z-1} y^* \right). \qquad (23)$$

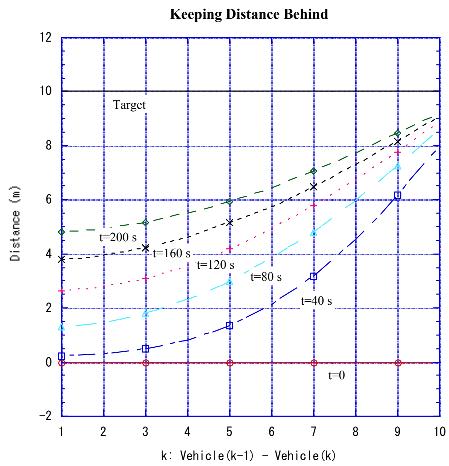

Fig. 8 Reference only to Trailing Car

Here is left z variables in gain part and this results in a non-uniform transition. Only the target interval information is provided from the last car propagates to the entire formation and takes a lot of time. This is recognized by Fig. 8 above. While the control law seems to possess the target interval information as $u_z = k(1-z)\left(y_z - \frac{z}{z-1} y^* \right)$, the reality is $u_n = k(y_n - y_{n+1})$.

5. DELAY EFFECT TO TRANSITION AND STABILITY

It was mentioned the strategy of

$$u_z = \frac{z}{z-1} k \left(y_z - \frac{z}{z-1} y^* \right) \tag{24}$$

provides a good transition. However, owing to the time delay existing in the loop prevents the formation from exhibiting a quick response. Collecting information between cars and deciding strategy at each car is done almost instantly., while the acceleration takes some delay Δt. That is the control is expressed practically by

$$u_z = e^{-\Delta ts} \frac{z}{z-1} k \left(y_z - \frac{z}{z-1} y^* \right). \tag{25}$$

The use of this with an first order approximation shows the response of

$$\dot{y}_z = -\frac{k}{1-k\Delta t} \left(y_z - \frac{z}{z-1} y^* \right). \tag{26}$$

The following condition is found obvious for the stability

$$1 > k\Delta t \tag{27}$$

and the resulted response time constant is found significantly affected by the delay. The stability here infers that the feedback should not be so high and sensitive and also that the acceleration performance of each car had better be higher. The latter is not always controllable and managing k is left tuned for the stability.

What needs to be stressed here is that collecting information and decision process should exclude delay. Provided there is delay Δt_c in obtaining information, from

$$u_n = u_{n-1}(t - \Delta t_c) + k(y_n - y^*) \tag{28}$$

it results in

$$u_n = \frac{z}{z-1} k \left(y_z - \frac{z}{z-1} y^* \right) \left(1 - e^{t/\Delta t_c} e^{-zt/\Delta t_c} \right) \tag{29}$$

and u_n itself may diverge. This will restrict the formation length and the size. The formation transition is affected by delays not only in acceleration but also in communication.

6. TWO DIMENSIONAL ORBITAL FORMATION CONTROL IN HILL'S MOTION

6.1 Formation Description in Hill's Motion

In the formation flying around an object, the equations of motion are written by Hill's popular equations. What is studied here is how the large scale formation consisting of many spacecraft should be controlled. As already mentioned, the strategy had better not rely on the high power communication equipment from resource point of view. And the centralized control gathering every information between the spacecraft may not be appropriate. Precisely speaking, bilaterally relayed communication may constitute such 'Fully Informed' formation, however, as the number of spacecraft increases, the communication traffic explodes. The subsequent study discusses the decentralized and teamed controlled strategy under the Hill's motion by extending one-dimensional formation control results.

The control law developed here is characteristic in terms of 1) decentralized control, and 2) isolated x-y motion. In actual applications, as the above equation shows, there is some time delay associated with eq. (43) that reconstructs the acceleration, besides the delay associated with the actuators' performance. As discussed previously, those delay is limited in terms of the stability and also may lengthen the settle time. And any delay in relay communication, even if it is small, will destabilize the formation and the formation size controlled is not infinite but limited.

Stationary Formation in Hills's Motion

So far the discussion has proceeded the station-keeping method for the formation intended to be fixed on the *x-y* coordinate. However, in this formation, without control acceleration, even when Δx_{ij}, Δy_{ij} are once controlled to zero, $\Delta x^{(2)}{}_{ij}$ does not become zero, oscillation remains in *x* direction. And the control acceleration needs to be applied continuously and it is not a practical formation except around some specific true anomaly region.

The subsequent discussion looks at the closed trajectory on the *x-y* coordinate in Hill's motion. Such locus appears when appropriate orbital parameters are taken.

Hill's popular equations of motion is written:

$$\ddot{x} - 2n\dot{y} - 3n^2 x = u, \quad \ddot{y} + 2n\dot{x} = v \tag{30}$$

In eq. (30), equating $x = x'$, $y = 2y'$ gives

$$\ddot{x}' - 4n\dot{y}' - 3n^2 x' = u, \quad \ddot{y}' + n\dot{x}' = \frac{1}{2} v. \tag{31}$$

This corresponds to a circular motion when trajectory is contracted 1/2 in y direction. And introducing the rotation coordinate defined as

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \xi \\ \eta \end{pmatrix} \tag{32}$$

makes the equations of motion converted on $\xi - \eta$ coordinate, where $\dot{\theta} = n$ always holds. Denote $p = \begin{pmatrix} \xi & \eta \end{pmatrix}^T$ and

$$\ddot{p} + A(\theta)\dot{p} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u \\ \frac{1}{2}v \end{pmatrix} = \hat{U} \quad (33)$$

is obtained for a spacecraft motion. Here,

$$A(\theta) = n\begin{pmatrix} 3\sin\theta\cos\theta & 1-3\cos^2\theta \\ -1+3\sin^2\theta & -3\sin\theta\cos\theta \end{pmatrix}. \quad (34)$$

Eq. (33) shows the steady state of $p = const.$, which implies $\xi = const.$, $\eta = const.$. And the stationary formation appears in the $\xi - \eta$ coordinate. Note this does not state the locus on the original *x-y* plane maintains a constant distance.

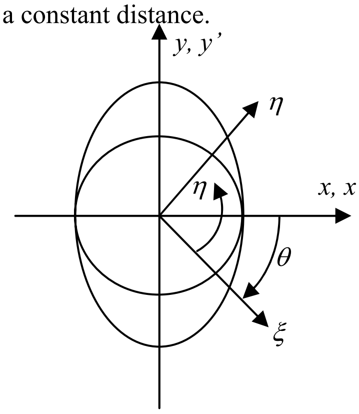

Fig. 9 Modified Coordinate

Stabilization of ξ–η Motion

The discussion starts from the stabilization of eq. (33) first. With scalar variables *c, k* , what follows proves the system (33) is stabilized. With the proposed control law example

$$\hat{U} = -c\dot{p} - kp, \quad (35)$$

when the Lyapunov function of

$$V = \frac{1}{2}\dot{p}^T\dot{p} + \frac{k}{2}p^T p \quad (36)$$

is taken, time derivative of it is written as

$$\dot{V} = -\dot{p}^T\left(c1 + A(\theta)\right)\dot{p}. \quad (37)$$

The eigen values of the matrix $A(\theta)$ are, regardless of θ, $\pm\sqrt{2}n$ and if

$$c > \sqrt{2}n \quad (38)$$

is satisfied, for any θ, eq. (37) becomes negative definite and the system is stabilized. Since *c, k* are all scalar, the control law proposed in eq. (35) admits the control to ξ, η direction independently.

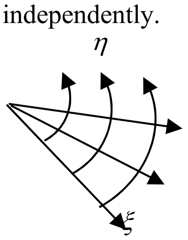

Fig. 11 ξ–η Coordinate

Formation Flying Control Laws

With all preparations, also assuming the uncoupled controller's structure to ξ, η directions, the relative distance to back-and-forth and left-to-right directions is expressed via z transform:

$$\Delta\ddot{p}_z + A(\theta)\Delta\dot{p}_z = \begin{pmatrix} \frac{1}{z_\xi}-1 & 0 \\ 0 & \frac{1}{z_\eta}-1 \end{pmatrix}\hat{U}_z \quad (39)$$

results and taking the advantage of uniform convergent control strategies developed through a single degree of freedom formation, the strategy of

$$\hat{U}_z = \begin{pmatrix} \frac{z_\xi}{z_\xi - 1} & 0 \\ 0 & \frac{z_\eta}{z_\eta - 1} \end{pmatrix}\left(c\dot{p} + kp\right) \quad (40)$$

is proposed here. This is rewritten to the control acceleration performed at each spacecraft as

$$\begin{pmatrix} u_z \\ \frac{1}{2}v_z \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{z_\xi}{z_\xi - 1}\left(c\Delta\dot{\xi}_{z_\xi} + k\Delta\xi_{z_\xi}\right) \\ \frac{z_\eta}{z_\eta - 1}\left(c\Delta\dot{\eta}_{z_\eta} + k\Delta\eta_{z_\eta}\right) \end{pmatrix}. \quad (41)$$

Naturally, the control acceleration is a cross feedback of both formation error in ξ, η directions. When the strategy is reversely converted to the time-domain, here is obtained

$$\begin{aligned} u &= \cos\theta\alpha_k + \sin\theta\beta_m \\ \frac{1}{2}v &= -\sin\theta\alpha_k + \cos\theta\beta_m \end{aligned}. \quad (42)$$

Here

$$\begin{aligned} \alpha_k &= \alpha_{k-1} + c\Delta\dot{\xi}_k + k\Delta\xi_k, \\ \beta_m &= \beta_{m-1} + c\Delta\dot{\eta}_m + k\Delta\eta_m \end{aligned} \quad (43)$$

appears. This structure is the accumulation structure that the uniformly convergent formation control concludes. Also as stated in one-dimensional formation discussion, it is, at the same time, a decentralized process that relays the accumulated information to the next, so that the entire formation is controlled with little delay.

7. CONCLUSION AND REMARKS

The paper presented the decentralized and localized formation keeping control strategies for both one and two dimensional systems. A new z-transformation approach was developed and successfully derived the relayed and decentralized control strategies. This dealt with first the formation decentralized control for the formation in Hill's motion.