# PRELIMINARY ANALYSIS OF LOW-THRUST GRAVITY ASSIST TRAJECTORIES BY AN INVERSE METHOD AND A GLOBAL OPTIMIZATION TECHNIQUE

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**ABSTRACT**

The design of interplanetary trajectories requires the solution of an optimization problem typically requiring a first guess solution. In addition the most recent development in low-thrust propulsion open the doors to new perspectives for the exploration of the solar system, while at the same time increasing the difficulties related to the trajectory design process. In this paper an automatic method for the preliminary definition of complex interplanetary transfer characterized by multiple swing by, either low-thrust or impulsive maneuvers is presented. The goal is achieved combining a novel methodology for the description of low-thrust arcs, based on an inverse method, with a global optimization algorithm based on an hybridization of an evolutionary search with a deterministic branching step. This approach allows a broad investigation of the solution domain as it is required in a preliminary design phases, where the availability a large number of first guess solutions is important.

## 1. INTRODUCTION

The design of an interplanetary mission is aimed at obtaining the maximum scientific benefit at the lowest cost; this deeply influences the definition of the trajectory. So far the concept of multiple gravity assist transfers has proven to be the best solution to this problem. Yet recently NASA Deep Space 1 and ESA Smart 1 showed the effectiveness of low-thrust systems as a primary propulsion, thus opening the doors to new missions in the solar system exploiting the beneficial effects of the combination of gravity assist manoeuvres and low thrust systems. Such a new scenario, characterized by new propulsion systems and driven by growing and challenging demands of exploration, makes the task of mission analysts even more difficult. Traditionally trajectory design has been accomplished mainly resorting to optimisation methods, based on gradient techniques or optimal control theory. These methods require a first guess solution, which deeply affect convergence, typically defined by the experience of mission analysts. Additionally the development of new design processes, mainly based on concurrent design engineering principles, as in the Concurrent Design Facility (CDF) in ESTEC, requires a quick assessment of a large number of solutions, possibly finding the best one. So far just a limited number of effective automatic preliminary design tools [1],[2],[3],[4] have been developed such as STOUR-LTGA by people from JPL and Purdue University, proving the importance of having an effective, sufficiently accurate and very rapid automatic tool capable of defining a large number of feasible preliminary solutions.

## 2. TRAJECTORY MODELLING

One of the major difficulty related to the preliminary design of low-thrust trajectories, is the unavailability of both sufficiently general analytical solution[1] and/or Lambert's like algorithms for a thrusting spacecraft. Consequently for most cases numerical propagation of the trajectory subject to a given thrust is necessary, with the solution of a NLP problem requiring a considerable computational effort. An interesting alternative, known as shape based approach, was proposed by Petropoulos and Longusky, consist of representing a trajectory linking two points in space with a particular parameterised analytical curve and then computing the required acceleration to follow the given trajectory. In this paper a novel approach based on the shape idea (here named inverse method), allowing a full consistent 3D description of low-thrust trajectories, is presented.

### 2.1 Low-Thrust arcs

Considering a given proper set of solutions $\tilde{\mathbf{r}}$, it is always possible to easily obtain, through algebraic computation, the necessary acceleration $\tilde{\mathbf{a}}_d$ of the propulsive system by:

$$\tilde{\mathbf{a}}_d = \ddot{\tilde{\mathbf{r}}} + \mu \frac{\tilde{\mathbf{r}}}{\tilde{r}^3} \tag{1}$$

however Cartesian coordinates, due to the large variations experienced even for Keplerian orbits, turn out to be unsuitable for the construction of simple shapes for a perturbed trajectory. A more accurate description of the evolution of a low-thrust trajectory can be done by the orbital elements; it is well known that the solution of the perturbed problems, in terms of these elements can be generally represented as:

$$p_i(t) = p_{0i} + \delta p_i(t) \tag{2}$$

where $p_{0i}$ is the initial condition of the i-th parameter, which remains constant if the motion is undisturbed, while $\delta p_i(t)$ is a time dependant function due to the effect of any perturbative action and which is typically small in magnitude for low thrust. Among all possible sets of elements, equinoctial modified parameters *[p,f,g,h,k,L]* have been chosen for their non-singularity and applicability to general orbits. Since low-thrust trajectories, likely with multi-spiral arcs, are going to be investigated, a better approach consists of representing the evolution of the equinoctial parameters in terms of the angular variable $L=\Omega+\omega+\nu$ [9]. Where ν is the true anomaly, ω is the anomaly of the pericentre and Ω is the argument of the ascending node.

Then supposing to have the shape of the trajectory, described by means of the chosen elements, it is possible to solve the inverse problem Eqn.1, in order to obtain the acceleration profile necessary to follow the imposed trajectory. The set of elements used to describe the evolution of the trajectory are here called pseudo-equinoctial elements $\alpha$ because they do not satisfy exactly Gauss' planetary equations for all times in the time domain, unless thrust is identically zero. Once the evolution of the position vector **r** is known in terms of pseudo-equinoctial elements, the velocity **v** and acceleration **a** can be computed by analytical differentiation:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr_j}{dL}\frac{dL}{dt} \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{dv_j}{dL}\frac{dL}{dt} \tag{3}$$

$$\frac{dr_j}{dL} = \sum_{i=1}^{5}\frac{\partial r_j}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial L} + \frac{\partial r_j}{\partial L} \quad \frac{dv_j}{dL} = \sum_{i=1}^{5}\frac{\partial v_j}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial L} + \frac{\partial v_j}{\partial L} \tag{4}$$

In order to obtain a set of pseudo-equinoctial elements that satisfies exactly the boundary conditions, the following nonlinear equations should be solved:

$$\mathbf{v}(\boldsymbol{\alpha}(L_0), L_0) = \mathbf{v}_0 \quad \mathbf{v}(\boldsymbol{\alpha}(L_f), L_f) = \mathbf{v}_f$$

$$\mathbf{r}(\boldsymbol{\alpha}(L_0), L_0) = \mathbf{r}_0 \,;\, \mathbf{r}(\boldsymbol{\alpha}(L_f), L_f) = \mathbf{r}_f \tag{5}$$

anyway for low values of the propulsive acceleration, like for low thrust engines, it is often sufficient to solve the easier linear problem for the boundary conditions of the pseudo-elements:

$$\boldsymbol{\alpha}(L_0) = \boldsymbol{\alpha}_0 \quad \boldsymbol{\alpha}(L_f) = \boldsymbol{\alpha}_f \tag{6}$$

For more accurate trajectories the solution of Eqn.6 is used as first guess for the solution of Eqn.5. Then, the control necessary to follow the imposed shape is algebraically obtained by Eqn.1, while the propellant mass ratio can be easily computed, as a fraction of the initial spacecraft mass. Due to change to the independent variable L a constraint relating the time of flight is added:

This approach is extremely fast and its computational cost low since no propagation is required and the differential problem is reduced to an algebraic one. Furthermore the use of orbital elements, easily allows a full 3D description, which turns out to be more suitable in order to evaluate the opportunity offered by low-thrust propulsion.

## 2.2 The Shape of the Trajectory

The inverse method requires the definition of the shape of a low thrust trajectory connecting two points in space. It is fundamental to define the proper family of parameterised shapes, which allows a suitable representation of a low thrust orbit. It has already been reminded that the general evolution of an orbital element is described by Eqn.2 with $\delta p_i(t)$ being the disturbing effect, which remains typically small in the presence of a low thrust perturbation. This may suggest that the effect of a low thrust can be treated, as a general perturbation, in terms of a secular and a periodical component, the latter having typically a net null effect averaged over the mean orbital period. This circumstance has driven to consider different possible shape for the evolution of the pseudo-equinoctial elements such as:

$$\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1(L - L_0) + \mathbf{p}\sin(L - L_0 + \varphi) \tag{7}$$

where p=$[p_1, p_2, p_3, p_4, p_5, \varphi]^T$ is a set of parameters shaping each pseudo-element, while $\boldsymbol{\alpha}_0$ and $\boldsymbol{\alpha}_1$ are related to the initial and final condition for each pseudo-element. Now the solution of Gauss planetary equations with an imposed thrust law dependent on the inverse of the square of the radius suggests an exponential trigonometric shape as:

$$\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 e^{\lambda(L-L_0)}\sin[\omega(L-L_0)+\varphi] \tag{8}$$

In this last case the frequency ω and the phase φare zero for the orbital parameter p, ω is common to all other orbital elements and the control parameters are $p=[\lambda_1,\lambda_2,\lambda_3,\omega,\varphi_1,\varphi_2]^T$. However if just the mean evolution of the elements is considered, the trigonometric term can be dropped and the shape parameters reduced to $p=[\lambda_1,\lambda_2,\lambda_3]^T$. It can also be noticed that, apart from the particular shape chosen, two free constants for each element $\alpha$ allow to satisfy the boundary conditions in terms of pseudo-elements, while the remaining free constants allow to meet additional constraints defined in the design process.

The inverse method does not provide necessary an optimal solution therefore any result should be considered as a feasibility assessment of a trajectory within a given level of thrust or other orbit's characteristics such as launch and arrival velocity, time of flight or a particular sequence of planets. It is anyway expected that as the shape of the pseudo-elements approaches the solution of the corresponding optimal control problem, the inverse method will yield the associated optimal control for the thrust.

### 2.3 Coast Arcs and Gravity Assist Manoeuvres

In order to have a preliminary design tool, capable of assessing different type of trajectories a model for coast arcs and for gravity assist manoeuvres, have been introduced. This allows to treat full ballistic or impulsive transfers or to consider coast arcs in LTGA transfer. If a pure coast arc between two planets is considered the initial condition is analytically propagated up to the time of a deep space manoeuvre is expected, and then a Lambert's algorithm is used to solve the problem between the deep space manoeuvre and the following planet. Swingby manoeuvres have been implemented with a 3D linked-conic model. The plane of the hyperbola, defined by the vector $\mathbf{n}_\pi$, can rotate around the incoming vector $\tilde{\mathbf{v}}_i$ of an angle $\eta$. The outgoing relative velocity is then computed rotating the incoming vector in the plane defined by $\mathbf{n}_\pi$, of an angle $\delta=\delta(r_p)$ where $r_p$ is the pericentre radius.

### 2.4 Composition of the Trajectory

The trajectory is divided into a number of phases equal to the arcs connecting the planets. The relative launch velocity is a free parameter and, in case of thrust arcs, the relative incoming velocity at the target planet is an additional free parameter. The relative outgoing velocity from each swingby is computed as explained in the previous section. Initial and final position are derived from the 3D analytical Ephemeris for each phase $k$, and the procedure is then iterated in the same way to all the phases. For multi-spiralling thrust arcs around the Sun, an integer auxiliary variable n is introduced. Therefore the final anomaly $L_f \in [0,2\pi]$, which defines the final position, is modified through: $\tilde{L}_f = L_f + 2\pi n$

Either fixed or free swingby sequence problems can be implemented. In the former case, given a sequence of gravity-assist bodies a search is started over a range of launch dates, encounter times and gravity assist parameters $\eta$ and $\delta$, in order to locate the best low-thrust gravity assist trajectory reaching a target planet. The solution is defined in terms of the most relevant design parameters such as: launch date, launch velocity (vector), transfer time to each planet of the sequence, arrival velocity at each planet of the sequence, altitude of the swing-by manoeuvre and thrust level required. Alternatively if the sequence is free a string of integer variables $n_b^i$ is added to the list of the parameters defining the trajectory, each integer variable $n_b^i$ representing the identifying number of a i-th celestial body.

The model presented above, in its general form allows the quite accurate description of impulsive multiple gravity assist interplanetary transfers and a preliminary definition of low-thrust trajectories. The complexity of the model increases the number of degrees of freedom and potentially optimal solutions. Moreover if the sequence is free the search for an optimal trajectory can be formulated as a general mixed integer nonlinar programming problem, which has been solved as explained in the following chapter. The complexity of the model can be easily reduced by a 2D representation of the trajectory or by removing deep space manoeuvres (only Lambert's arcs are allowed between two planets) and the parameters $\eta$ and $\delta$. In the latter case swingbys need to be propelled (Δv-GA) and the total cost of the Δv-GA has to be minimised. Notice that this reduced model decreases the degrees of freedom and simplifies the search but rules out a number of potentially interesting options such as resonant trajectories [7].

## 3. THE OPTIMISATION APPROACH

The proposed optimisation approach, implemented in software code called EPIC, is based on Evolutionary-Branching (EB)[5,6]. Evolutionary-Branching is a hybrid deterministic-stochastic approach to the solution and characterisation of constrained and unconstrained multimodal, multivariate nonlinear programming problems with mixed integer-real variables and discontinuous quantities. The EB approach is based on the following principal ideas:

- An evolutionary strategy is used to explore globally and locally the solution space $D$. Then a branching scheme, dependent on the findings of the evolutionary step, is used to partition the solution domain in subdomains. On each subdomain a new evolutionary search is performed. The process continues until a number of good minima and eventually the global one are found.
- The search is performed by a number of agents (explorers): each solution $\mathbf{y}$ is associated to an agent. and is represented by a string, of length $n$, containing in the first $m$ components integer values and in the remaining $s$ components real values. This particular encoding allows the treatment of problems with a mixed

integer-real data structure. A hypercube **S** enclosing a region of the solution space surrounding each agent, is then associated to **y**. The solution space is then explored locally by acquiring information about the landscape within each region **S** and globally by a portion of the population, which is continuously regenerated forming a pool of potential explorers.

- Each agent can communicate its findings to the others in order to evolve the entire population towards a better status.
- During the evolutionary step a discoveries-resources balance is maintained: a level of resources is associated to each agent and is reduced or increased depending of the number of good findings of the agent.

Evolution is governed by four fundamental operators: mutation, migration, mating and filtering. The mutation operator generates a new individual perturbing an old one. The mating procedure takes two individuals and generates two children mixing the genotypes of the two parents. Four schemes are used to mate individuals: single point crossover, arithmetic crossover, extrapolation, by which a new individual is generated on the side of the best individual between the two parents, and a novel operator called *second order extrapolation*. This mating operator uses two parents and the child generated with an extrapolation mating to build a local quadratic model of the fitness function and then takes the minimum of the quadratic form.

### 3.1 Environment Perception and Migrations

Each region **S** is evaluated using a mechanism called *perception*. This operator samples the environment in order to improve the status of the agent. A new region **S** is then associated to the best discovery resulting in a migration of the explorer towards a place where better resources are expected. For this reason each hypercube **S** is here called *migration region*. The subpopulation is generated with the following procedure: a first child is generated, within **S,** mutating the parent, then an extrapolation mating is performed. The two resulting children and the parent are then used to generate a third child using second order extrapolation mating. The procedure is repeated until either an improvement of the agent is found or a number of samples equal to the number of coordinates have been generated. The contraction or expansion of each region **S** is regulated through a parameter $\rho$ which depends on the findings of the perception mechanism: if none of the samples is improving the agent's status, the radius is contracted taking the distance of the best sampled point from the agent.

### 3.2 Filtering & Ranking

A permanent population of *npop* agents is maintained from one generation to another. Each individual has a chance to survive and to become an explorer provided that it remains inside the filter. The filter ranks all the individuals on the basis of their fitness from best to worst. All the individuals within the filter are explorers the others belong to the above mentioned pool and are either hibernated (i.e. no operator is applied) or mutated while migration is applied to all individuals within the filter. The probability of being mutated or hibernated depends on their ranking. Mating is operated between all the individuals that present an improvement during one generation (exchange of information).

### 3.3 Regeneration

The mating operator is used also to prevent crowding of too many agents in the basin of attraction of the same solution: if many agents are intersecting their migration regions and their reciprocal distance falls down below a given threshold, a repelling mechanism is activated which mates the worse individual with the boundaries of the subdomain $D_l$. The threshold is a function of the number of crowding agents and a best-in-worst-out rule is used to select the repelled agent.

### 3.4 Branching Step

Branching is based on the output of the evolutionary step in two ways: the cutting points for the partition of the domain $D$ are either selected according to an inference model (predictive branching) or taking the worst and the best individuals out of the evolutionary step; for each subdomain the density of agents $\varpi_{D_l}$ and the average value of the findings $\varphi_{D_l}$ are computed. The subdomains are then ranked according to the quantity:

$$\psi_{D_l} = \sigma\varpi_{D_l} + (1-\sigma)\varphi_{D_l} \tag{9}$$

where $\sigma$ is the weighting factor used to favour either exploration or convergence and the best, according to (9), among the subdomains for which something better has been discovered, is subdivided further.

### 3.5 Constraint Handling Technique

At each evolution step the population of solutions is divided into two subpopulations and a different objective function is assigned to each one, namely one subpopulation aims at minimising the original objective function while the other aims at minimising the residual on the constraints defined as:

$$\min_{y\in D} F(\mathbf{y}) = \sum_{j=1}^{q} e^{R_j} \tag{10}$$

where $q$ is the number of violated constraints and $R_j$ is the residual of the j-th violated constraints. The two subpopulations are coevolved in parallel and individuals are allowed to move from one population to the other.

Moreover, if $F^*$ is the value of the fitness function of an agent **y** inside the feasible set, the fitness function of a new individual, or a sample, generated from **y** is augmented in the following way:

$$\min_{y \in D} F = \begin{cases} F^* & \text{if every } R_j \leq 0 \\ F^* + \max R & \text{if any } R_j > 0 \end{cases} \tag{11}$$

The described strategy co-evolving two populations with two different goals, allows a flexible search for feasible optimal solutions: in fact through the described use of the perception mechanism feasibility can be enforced on all feasible solutions or just on a subset among the feasible ones. In the latter case a more extensive search along the boundary of the feasible region is allowed, while preserving the feasibility of at least the best solution,.

### 3.6 Mixed Integer nonlinear programming approach

IMAGO allows to leave the sequence of planetary encounters completely free. This leads to the solution of an integer nonlinear programming problem. The sequence of encounters is implemented as a string of integer numbers $s_i=[n_b^1, n_b^2, \ldots\ n_b^m]$ where $n_b^j$ is the identifying code of a planet and $m$ is maximum number of planetary encounters. The branching scheme is then applied also to the integer part of the vector partitioning the set of possible integer values assumed by the parameters $n_b^j$ . At the evolution level the mixed integer-real vector is processed evolving the integer part at a different rate with respect to the real part, i.e. for each generated sequence of planets the a number of sampling steps of the real subdomain are taken. This allows to partially avoid a typical barrier effect occurring in the automated design of multiple gravity assist manoeuvre with free sequence: when a good solution is found for a given set of launch date, time of flights and date of planetary encounters, any variation of the sequence that preserves the set causes a drastic increase in the fitness function.

## 4. TEST CASES

The methodology presented in the previous chapter is the core of an integrated automatic design tool called IMAGO (Interplanetary Mission Analysis Global Optimization), capable of either quickly or deeply investigating large domains searching for the most promising trajectories.

### 4.1 Direct Transfer To Mercury

A direct transfer to Mercury, due to its eccentricity high inclination and high Δv requirements, represents the best example to test the capability of IMAGO in terms of shapes, multiple revolution and 3D description. Launch and arrival relative velocity are set equal to zero and the maximum acceleration is constrained to 5e-4 m/s$^2$, while the $I_{sp}$=3200s and the maximum time of flight is 1400 days. Then EPIC has been applied to this case bounding the departure date within the interval [3285 5475] and the number of revolutions within the interval [1 6].

The optimiser found a number of optimal solutions, among them the equinoctial parameters of one of the best have been plot in Fig.1 and compared to the shaped parameters. As can be seen the shaped parameters follow the mean value of the osculating parameters. Fig.2 demonstrates how the thrust remains always below the imposed constraint while the inplane (elevation) and out-of-plane components of the thrust are reported in Fig. 3. Notice how the out-of-plane component oscillates with once per orbit with maximum amplitude when the spacecraft is far from the Sun. As a further validation of the result the solution obtained with IMAGO has been used as first guess for a refined optimisation with the software DITAN[8]. As can be seen in Tab.*1* the two results are in good agreement. Notice that the first guess produced by IMAGO is in general an overestimation of the actual cost of the transfer.

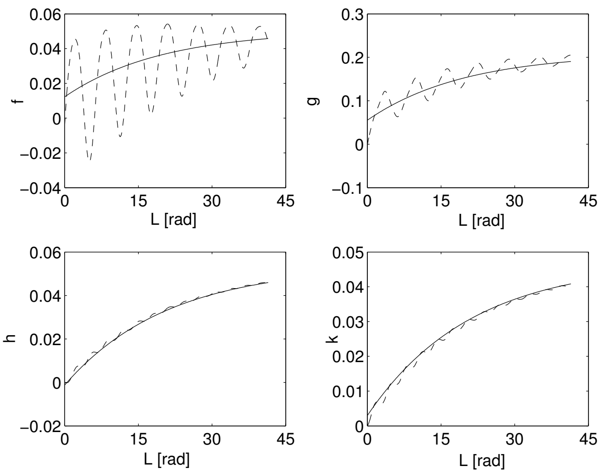


Fig. 1. Shaped and propagated osculating elements: solid lines represent shaped parameters

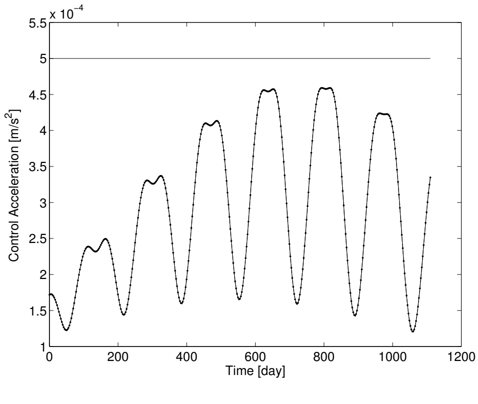

Fig. 2. Control Acceleration Profile

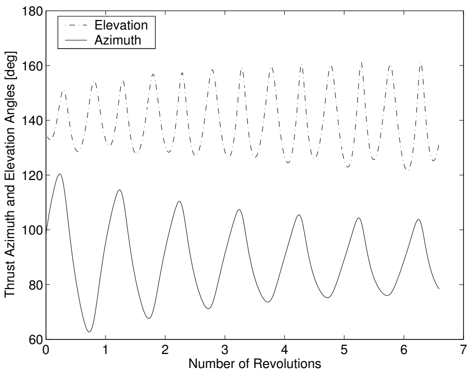


Fig.3 Azimuth and Elevation thrust angles

*Table 1. Summarising transfer information*

| Software | Launch Date | TOF (day) | Prop. mass ratio |
|---|---|---|---|
| IMAGO | 25/02/2014 | 1109.95 | 0.534 |
| DITAN | 18/04/2014 | 1084.44 | 0.515 |

### 4.2 Ballistic Free Sequence Transfer to Jupiter

An interesting advantage offered by the proposed optimisation approach to find efficient transfer using multiple swing-bys (MGA) and deep space manoeuvres, without prescribing a sequence a priori. This is a relevant application of IMAGO since these kinds of solutions typically turn out to be extremely interesting even for a possible low-thrust option.

A number of preliminary solutions for the optimal transfer to Jupiter have been designed performing the search over the interval [3650 10950] MJD and considering the possibility of a maximum of 3 encounters with Venus, Earth and Mars before the arrival at Jupiter. The departure velocity have been constrained to stay below 5 km/s.

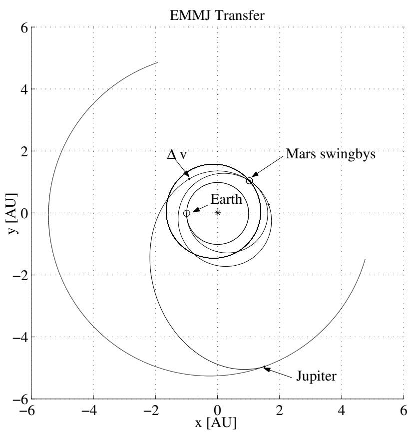


Fig. 4. Earth-Jupiter transfer via Mars

Notice that, although the number of degrees of freedom introduced by that the model implemented in IMAGO makes the search extremely challenging, it allows to have several alternative realistic solutions including resonant orbits as for the EMMJ case reported in Tab. 2.

*Table 2. Examples of MGA transfers to Jupiter*

| Sequence | EVEEJ | EVEEJ | EVEEJ | EMMJ | EVVEJ |
|---|---|---|---|---|---|
| LaunchDate | 17/08/10 | 29/07/10 | 27/10/13 | 20/03/14 | 28/10/13 |
| $C_3$(km/s$^2$) | 8.41 | 11.6 | 13.659 | 10.95 | 15.01 |
| $\Delta V_{tot}$(km/s) | 2.1 | 1.6 | 0.1347 | 3.10 | 0.618 |
| TOF (days) | 1895.5 | 2685 | 2082.0 | 2243 | 2404.3 |
| $V_\infty$(km/s) | 5.64 | 5.5 | 5.798 | 4.64 | 6.305 |

In the same table other alternative transfers are presented, some with a low C3 and a high total cost of the deep space manoeuvres $\Delta V_{tot}$, others almost ballistic.

### 4.3 Low-thrust transfer to Jupiter.

For the last test case a comparison approach with the tool STOUR-LTGA has been followed [4], thus showing the capability of IMAGO in treating a complex LTGA transfer with two flybys. The sequence EVEJ has been investigated in the year 2015 and a thrust-coast arc has been set after the Earth flyby .

*Table 3. Comparison of IMAGO and STOUR-LTGA*

| | IMAGO | STOUR-LTGA |
|---|---|---|
| Launch Date | 14/04/2015 | 09/05/2015 |
| Launch $V_\infty$ | 1.8 km/s | 2 km/s |
| Earth-Venus TOF | 184 days | 119 days |
| Venus flyby alt. | 8816 km | 4481 km |
| Venus-Earth TOF | 386 days | 345 days |
| Earth flyby alt. | 5964 km | 4219 km |
| Earth-Jupiter TOF | 901 days | 1027 days |
| Arrival $V_\infty$ | 4.53 km/s | 5.97 km/s |
| Prop. Fraction | 0.48 | 0.485 |

IMAGO located an interesting solution for this transfer with a smaller arrival velocity, but with comparable propellant consumption, thus showing the capability of treating complex low-thrust gravity assist trajectories.

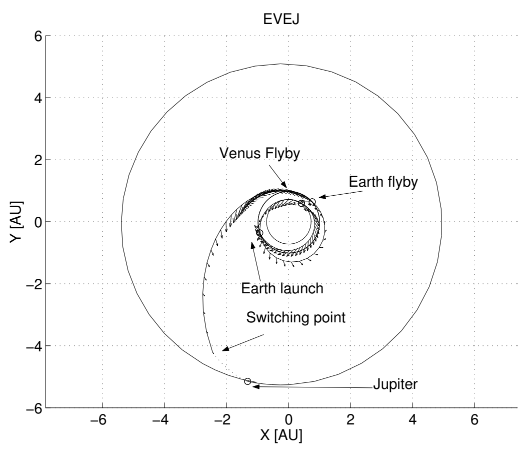


Fig. 5. Earth-Venus-Earth-Jupiter LTGA transfer

## 5. CONCLUSIONS AND RECCOMENDATIONS

In this paper a preliminary design tool for interplanetary trajectories has been presented and its capability of dealing with low-thrust direct transfer, LTGA and multiple swing-by ballistic trajectories has been proved. The various test cases show how this tool can either quickly assess sub-optimal solutions or widely and exhaustively investigate large domains thus providing an effective tool for preliminary mission analysis of interplanetary missions. Such goal has been mainly achieved thanks to the combination of thorough simplified model of the physics of the problem, with a particular global optimisation technique which exploits the suitability of evolutionary methods and the strength of the branching step, which permits the an exhaustive investigation of large and multi-modal domains. As a result there is a strong suggestion that evolutionary algorithms are suitable for preliminary trajectory design and strongly competitive with respect to systematic search.

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