COMPARISON OF ANGLES ONLY INITIAL ORBIT DETERMINATION ALGORITHMS FOR SPACE DEBRIS CATALOGUING

Fran Martinez Fadrique, Alberto Águeda Maté, Joan Jorquera Grau, Jaime Fernández Sánchez, Laura Aivar García
GMV, Isaac Newton 11, 28760 Tres Cantos, Spain, +34 91 807 21 00
e-mail: **fmartinez@gmv.com**, **aagueda@gmv.com**, **jjorquera@gmv.com**, **jfernandez@gmv.com**, **laivar@gmv.com**

Abstract: *The constantly increasing growth of the space debris population is also causing that more and more devices are looking into the sky in search of undetected objects. The process of orbit determination and further object cataloguing requires the initialisation of the object orbital state. This process is particularly complex in the cases when only angular observations from passive devices are available (e.g. topocentric right ascension and declination from a ground telescope). This paper describes the process of initial orbit determination when only a limited number of angular observations are available. Different orbital scenarios (e.g. LEO, MEO, GEO) are analysed together with the available algorithms. The analysis focuses on the suitability of the algorithm for the different orbital regimes and also in the robustness of the solution. The main objective of the analysis is to evaluate the adaptation of the algorithms and their parameterisation for the implementation in operational automated scenarios.*

Keywords: initial orbit determination, angles only, space debris cataloguing

1 Introduction

The constantly increasing growth of the space debris population is also causing that more and more devices are looking into the sky in search of undetected objects. In the process of collecting object ephemeris it is necessary to initialise the ephemerides of those objects detected for the first time. This process is essential to permit that this first detection materialises in the possibility to further track the object to improve and maintain the accuracy of the objects ephemerides for cataloguing and collision assessment purposes.

The process of initial orbit determination of non-collaborative objects can be categorised depending on the availability or not of the measured range from the detector to the object.

When the range is available the initialisation of the orbital ephemerides is to some extent simple already starting from just two observations. (Lambert method) or with improved performance using a third observation (Gibbs and Herrick-Gibbs methods). These algorithms perform very well when the observations are close to one another (i.e. small time separation without complete orbital revolutions in between) This is the common situation in space surveillance and tracking scenarios allowing to unambiguously estimate the initial orbit state.

Of greater interest, mainly due to its complexity, is the case when no range is available. Survey of high altitude orbits is normally performed using optical devices that provide only angular measurements from which the initial orbital state has to be derived. Several algorithms have been provided in the literature addressing this problem in an attempt to provide a solution with sufficient accuracy. The essence of the angle only methods is to estimate an approximation of the range from the geometry of at least three sets of angular observations (i.e. topocentric right ascension and declination or azimuth and elevation) and a simplified dynamical model for the object motion. Once the range is available the initial orbit determination problem reduces to the Herrick-Gibbs problem.

The purpose of this paper is to review these algorithms, analyse their performance and what is most important, analyse their robustness to provide reliable solutions.

The most classical algorithms addressing the angles only initial orbit determination are the Gauss and Laplace methods. These two methods provide comparable results for three sets of angular observations that are not far apart in time; this is a common tracking scenario when an object is

initially detected during a survey and further observations are acquired in subsequent tracking sessions a few minutes apart. Similarly to Gauss and Laplace, Gooding proposes a more sophisticated algorithm that allows for several possible solutions also starting from a set of three angular observations.
These methods, like many others found in the literature, provide a solution for the initial orbit determination problem, but to which extent is it possible to rely that the provided solution actually represents sufficiently the actual trajectory of the object? The answer to this question has been found to be more difficult to answer that it initially seems. When analysing all these methods in an evolving orbit determination scenario (e.g. a whole pass over a ground station for a medium altitude satellite sampled regularly at one minute intervals) it is easy to identify areas where the algorithm becomes singular and the provided solution deviated from the true solution to unacceptable levels of error. Adding more observations to the process mitigate partially the singularity problem but in scenarios where a limited number of observations is available one cannot rely on this benefit; in any case the singularity is not fully eliminated.
In the analysis conducted in this paper other algorithms that use a fourth set of angular observations to reduce the singularity have been analysed. The conclusion is that the singularity cannot be fully eliminated but its effect can be truly mitigated to level comparable to the results in the non-singular geometries. The effect of the equations required to deal with this fourth observation also introduce noise in the solution for non-singular geometries what introduces the necessity to analyse the problem to detect when the geometry is singular to adequately use the equations such that the optimal solution is always attained. The method by Baker-Jacoby has been analysed in detailed in the context of mitigating the effect of the geometrical singularity.
In an operational scenario all that has been mentioned above leads to the necessity to establish the level of reliability of the estimated initial state. The conducted analysis on this respect addresses the situation when just three observations are available, the means to identify from these three observations whether the geometry is singular or not and how to apply the algorithms adequately when the forth observation is available. Results from several orbit determinations scenarios (LEO, MEO, HEO and GEO) are provided and performance comparisons provided.

2. Objectives

This paper focuses in the analysis of orbit determination scenarios based on angular observations only in the extreme when just a few observations (three in the limit) are available. Whereas the orbit determination with a collection of observations can be implemented with sufficient accuracy using a classical least squares method, it is not unusual that after a detection of a new object just a few observations are available that make the least squares approach not applicable. Vallado in [1] defines the problem of initial orbit determination as the successful computation of a initial state that permits the further processing of observations in a traditional orbit determination scheme (e.g. least squares or Kalman). Whereas this can be consider as sufficient definition for the initial orbit determination success it is still necessary to guarantee that subsequent observations can be identified as associated to the same object such that the least square process can be implemented.
This paper analyses the case of a limiting scenario when just 3 or 4 observations are available for orbit determination. The step into the least squares is then only feasible if the propagation of the orbit permits that observations from the object can be obtained at future epochs with guarantee that correlate to the intended object, i.e. the telescope can be pointed to the point in space where the object is expected to be.
The analysis in this paper is then parameterised according to the following aspects:

- Different orbital configurations, GEO, MEO, LEO, HEO
- Applications and comparisons of different algorithms, Gauss, Gooding, Baker-Jacoby
- Analysis of observation separation

3. Orbital configurations

Scenarios where initial orbit determination based on angular measurements is applicable need to be identified and characterised. A fair assumption to be made is that the observations come from an optical device (telescope) located on the Earth surface (the analysis can then be easily extended to an optical device on orbit around the Earth). Traditionally the telescope observations have been used to survey and follow-up objects in the geostationary region. The follow-up of objects in the lower orbital regimes (MEO and LEO) is usually performed by radar means although recent studies have also analysed the possibility to use telescopes to survey regions in MEO and LEO altitudes.

The following scenarios have been considered as a sufficient representation of typical orbital scenarios suitable for angles-only initial orbit determination:

- Geostationary orbit (GEO) at longitudes 45ºW, 30ºW, 0º 30ºE and 45ºE. The main purpose to analyse the effect of longitude (relative to the observing site) in the orbit solution
- Mean altitude orbit (MEO), circular with semimajor axes of 32000, 22000 and 12000 km and 56º of inclination.
- High eccentric orbits (HEO)
    - typical geostationary transfer configuration (GTO) with semimajor axis of 24500 km, eccentricity of 0.72 and coinciding lines of apses and nodes.
    - Molniya orbit with semimajor axis of 26550 km, eccentricity of 0.72 and 63.5º of inclination.
- Low Earth orbits (LEO)
    - Sun-synchronous orbit at an altitude of 800 km
    - Typical altimetry mission at an altitude of 1120 km 65º of inclination

The effects of the latitude are introduced in the problem considering the telescopes in two different locations, continental Spain and the Canary Islands. These two sites represent two intermediate latitudes where telescopes are commonly located. The two places have been selected with similar longitudes to decouple the effect of longitude for the objects in the GEO region. The effect of longitude is just applicable to the objects in the GEO due to the different line of sight depending on the relative position (azimuth and elevation) with respect to the observing site; for objects in all other orbital regimes the objects will basically take all possible combinations of azimuth and elevation if sufficient passes are considered.

The propagation of these orbits is implemented using a detailed numerical orbit propagator. This guarantees that any dynamical effect that, through the orbit, may have an effect in the performance of the initial orbit determination is taken into account.

4. Tracking data simulation

The analysis shown in this paper is based on simulated observations. The availability of real data is limited, in particular for non GEO objects. The objective of the analysis is to perform an exhaustive survey of all configurations and therefore the uniform availability of data is necessary, this discards the possibility of using real data in a generalised manner.

True simulation of optical observation should take into account illumination and atmospheric conditions. If this is literally taken into account the amount of observations is limited and can only be increased by analysing a wider number of scenarios (e.g. increase the granularity in the longitudes for the GEO objects) or extending the time period for which measurements are generated. To mitigate this, the illumination aspects have been removed from the simulation and observations are generated at all epochs when geometry permits, i.e. the object is accessible by the telescope field of view. The atmospheric effects are retained by application of adequate tropospheric models for optical observations (e.g. Murray-Marini)

Characterisation of the generated tracking data is to be consistent with configurations that can be expected in real operations. As one of the objectives is to analyse the optimal distribution of data different data rates are simulated. Two aspects are then analysed: the sensitivity of the results to the separation of the 3 or 4 available observations and then the optimisation of data separation to obtain the best orbit determination. This should lead to the association of orbital scenario and observation acquisition such that it is possible to program the follow-ups after initial detection such that the performance of the cataloguing of the object is optimal through the best possible initial orbit determination accuracy.

5. The angle-only initial orbit determination algorithms

In the literature there are several approaches to the formulation of the initial orbit determination based on a minimum number of angular observations. The problem is described in detail in [2], [3], [4], [5] and [6]. From the documented algorithms the following have been selected

- Gauss: classical and simple algorithm that can be used as reference and whose performance is documented
- Gooding: a more sophisticated algorithm that is used in more modern approached to this problem
- Baker-Jacoby: uses a fourth observation to mitigate the coplanar singularity present in the other algorithms.

As shown later, the coplanar singularity appears mainly in the processing of MEO objects. This is one of the areas of most interest because the use of telescopes for the survey of the MEO region is already being explored. Then the selection of the Baker-Jacoby algorithm intends to address the possibility of having a more robust initial orbit determination for this particular scenario.

Before entering into the numerical analysis and the comparison of the algorithms and scenarios it is necessary to look at the potential performance that one could expect. There are essentially two sources of error

- Error associated to the weakness of the angular measurements in the orbit determination process. This error appears even if the observations have no noise and is the consequence of the poor geometry used to compute the range from the angular values. This is most critical for the GEO objects.
- Error associated to the noise in the observations. For observations to a GEO object separated 1 minute, a level of noise in the observations of 0.001º represents 0.4% of the angle swept between the observations and may lead to an error of 170 km in semimajor axis.

The approach taken by the selected algorithms are based mostly on the adjust of a simplified orbit model through the observations; this lead to differences between the computed lines of sight and the actual ones reported by the measurements (already in the case of observations with zero noise). Osculating effects and high frequency perturbations lead to differences that are not accounted for in the initial orbit determination process. All in all the problem is expected to be poorly conditioned and small perturbations in the input could be amplified in the estimation of the output orbital state.

All algorithms for initial orbit determination based in angles rely on the estimation of the range that matches the dynamics and the input angular observations. Therefore the state vector achievable accuracy is a function of the accuracy with which the range can be obtained. The analysis in the following section is then implemented in terms of estimated range. Once the range is computed the orbital state is computed using Herrick-Gibbs which is common to all three algorithms; it can be assumed that to equivalent range accuracies equivalent state vector accuracies can be achieved. At the end of the comparison an analysis of the orbit state estimation capabilities is provided for the bets initial orbit determination scenario.

6. Algorithm Verification

The same scenarios used for the final analysis are used first to verify that the implementation of the algorithms is correct. This verification is performed using simulations without noise. Figure 1 below shows the comparison of the initial orbit determination performed on this zero noise data; the represented magnitude is the error of the range estimated from the series of angular observations in groups of three (Gauss and Gooding) or four (Baker-Jacoby) taken every minute. The implementation of the three algorithms looks consistent to the expected levels of accuracy. These results also represent the maximum accuracy that can be obtained by any of the methods.

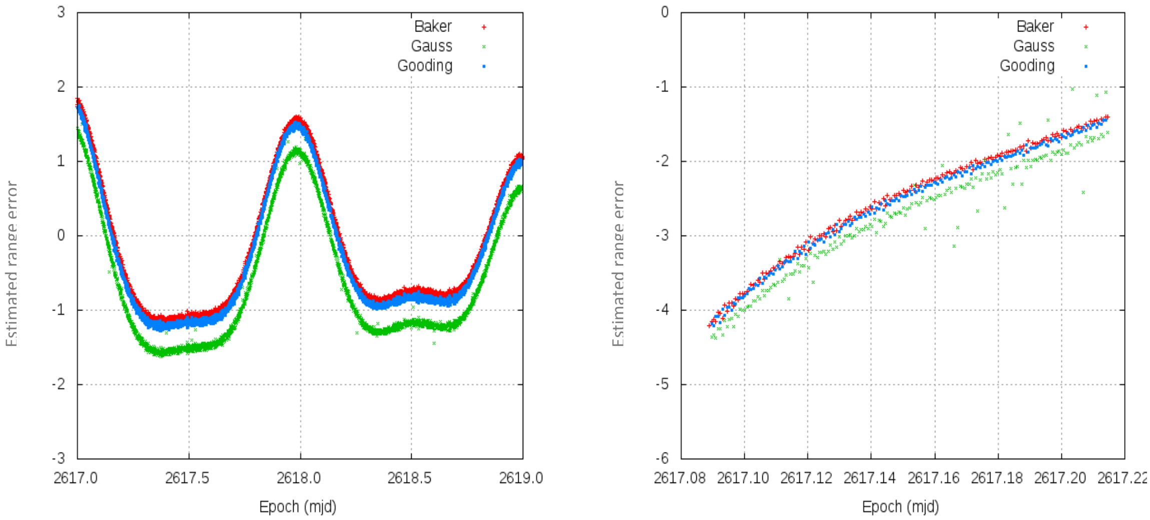

Figure 1. Zero noise performance for GEO (left) and MEO (right)

Additionally, verification on the internal consistency of the algorithms has been preformed. This check relates to the stability with respect to the separation of the observations. Figure 2 shows the results of each of the algorithms for observations separated by 1, 2, 4, 8 and 16 minutes.

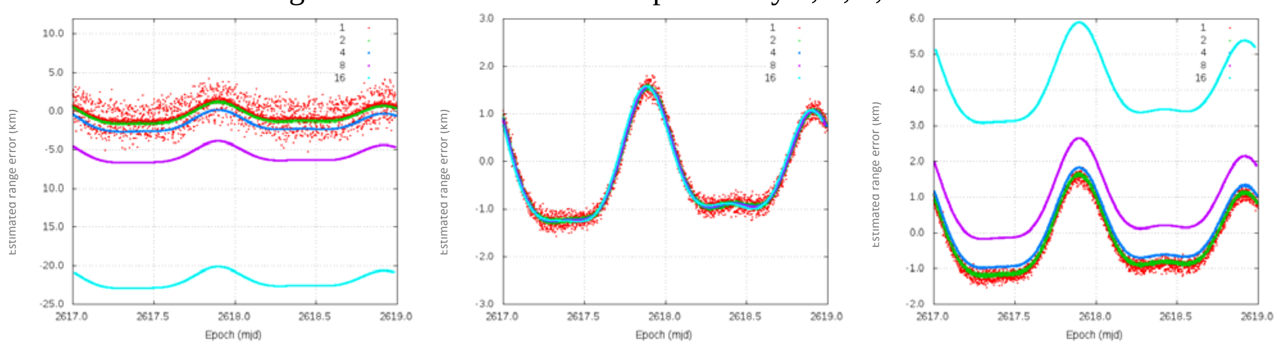

Figure 2. Zero noise performance for Gauss (left), Gooding (middle) and Baker (right)

There are two dependencies with respect to data separation that can be observed

- The noise (introduced by the models since the observations data noise is zero) is reduced as the separation increases. This is particularly noticeable for the separation of 1 minute in all three algorithms.
- The mean solution is only consistent for the Gooding algorithm. Gauss and Baker-Jacoby show a bias that depends on the observation separation. The solution can be considered as unbiased for separations up to four minutes.

7. Initial orbit determination scenarios

The following sections show the performance of the different algorithms for the selected configurations, data separations and algorithms. In all cases the data is simulated with different levels of noise corresponding to the total error in the observations generation from all possible sources; this is represented as a Gaussian distribution with respect to the observation series. Although this is clearly a simplification with respect to reality and because the objective is to identify the accuracy and robustness of the algorithms with respect to observation errors this approach should be sufficiently representative. The number of observations in each data set is just 3 or 4 and therefore the Gaussian distribution guarantees that all possible error combinations between the observations in the set is analysed when a sufficiently large number of sets is processed.
The reference noise for the observations is 0.0001° assuming that state of art telescopes should lead to that level of accuracy or better. Atmospheric effects, systematic pointing errors and other errors associated to the processing of the telescope images leading to systematic errors are analysed separately.

7.1 Geostationary orbits

The following figures show the initial orbit determination from all three algorithms on an object at 30ºE from the telescope in continental Spain; each plot contains the data at different observation separation of 1, 2, 4, 8 and 16 minutes.
The overall behaviour is similar in all three cases and the dispersion from one epoch to the next decreases as the separation increases. This behaviour could be expected since the relative weight of the noise decreases with the separation. Gauss presents a uniform behaviour while Gooding and Baker-Jacoby produce a number of degenerated solutions for small separations (1 and 2 minutes) that correspond to hyperbolic trajectories consistent with the geometry. These degenerated solutions did not appear in the zero noise what means that they are associated to the uncertainty introduced by the measurement noise.

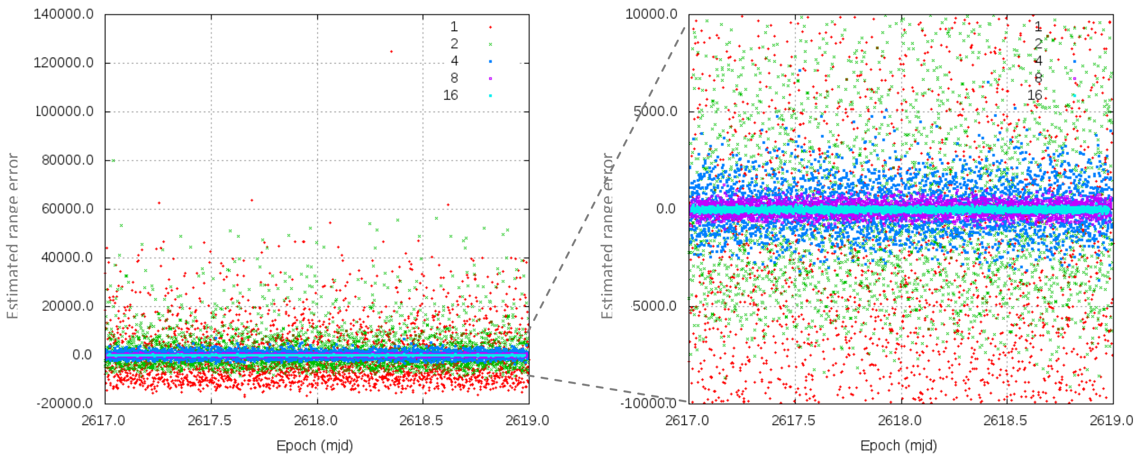

Figure 3. Gauss on geostationary orbit at 30ºE

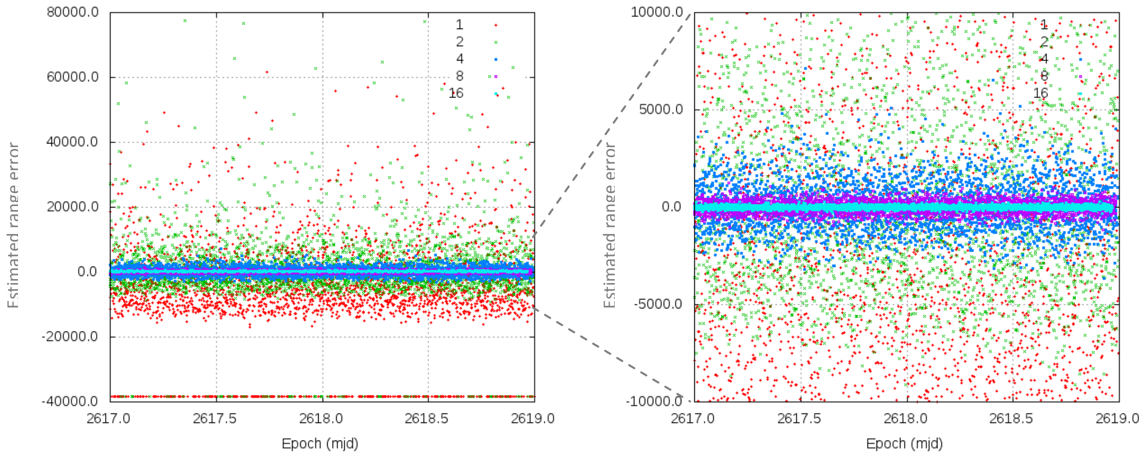

Figure 4. Gooding on geostationary orbit at 30ºE

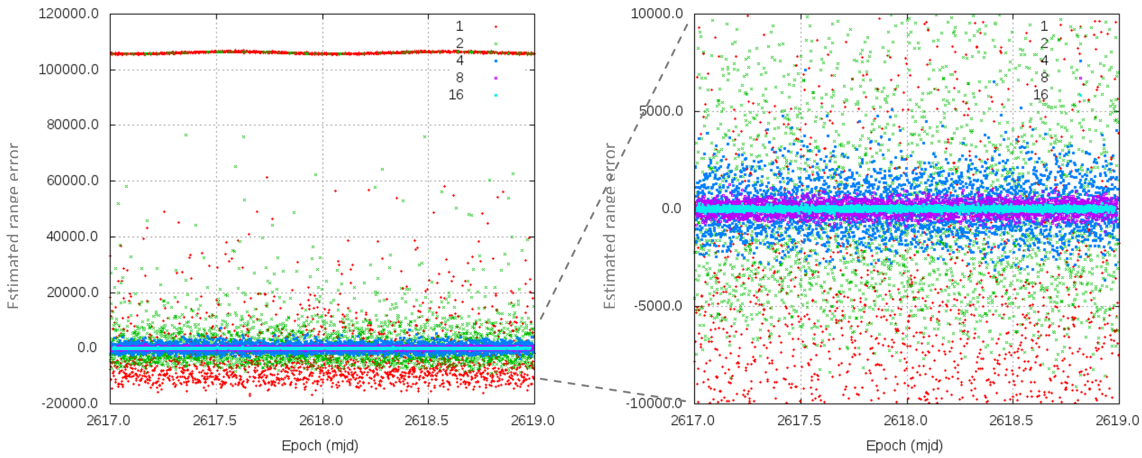

Figure 5. Baker-Jacoby on geostationary orbit at 30ºE

Although the previous plots provide a good insight in the overall behaviour of the process it is necessary to quantify the relative performance of the algorithms. Table 1 shows this comparison where the best results in each category is highlighted in red. The optimal measurement separations happen for Gauss between 8 and 16, for Baker-Jacoby between 16 and 32 and the best solution is obtained with Gooding with a separation of 32 minutes.

Table 1. Solution dispersion as function of observation separation

Separation (minutes)	Gauss		Gooding		Baker-Jacoby	
	Mean (km)	Sigma (km)	Mean (km)	Sigma (km)	Mean (km)	Sigma (km)
1	1998.070	77930.655	18426.816	272469.400	28421.270	46051.056
2	2100.744	8018.455	2019.732	10917.478	3640.262	15346.947
4	96.190	1279.882	96.660	1277.452	97.823	1279.931
8	0.097	308.943	5.604	308.823	6.607	308.994
16	-21.636	74.362	0.151	74.775	4.375	74.410
32	-87.213	18.541	-0.346	18.965	16.988	18.590

The remaining analysis is the dependency with the longitude of the object with respect to the observing site. This is shown in Fig 6. where it can be observed that the dependency with longitude is minimum. The data shown corresponds to the separation of 16 minutes.

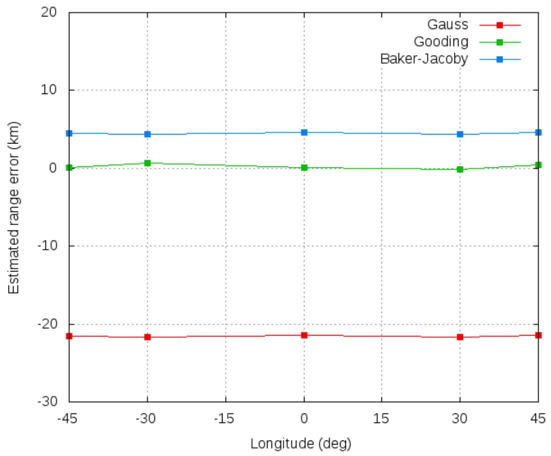

Figure 6. Algorithm longitude dependency

7.2 Excentric orbits

Two types of eccentric orbits are considered in the following two sections, geostationary transfer orbit and Molniya

7.2.1 Geostationary Transfer Orbit (GTO)

The following figures show two passes of a geostationary transfer orbit for measurement separations between 1 and 16 minutes for the three analysed algorithms. Gauss and Baker-Jacoby present very similar behaviours with degradation at low elevations; Gooding maintains the performance even at those low elevations. Like in the geostationary case, the performance with small separations is very much degraded by the presence of noise. The plots on the right represent the same estimated range error but as function of true anomaly. Performance is worse around apogee, particularly for small separations, because the dynamics are poorer. It is to be noted that GTO normally have the Sun in the apogee direction and therefore this area cannot be observed while perigee is in eclipse; one can expect to have observations during limited periods before and after perigee where the performance is intermediate.

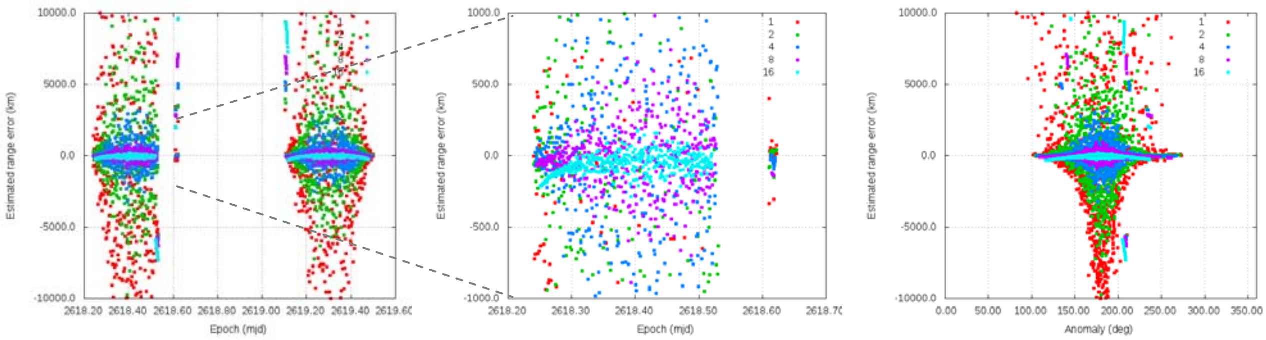

Figure 7. Gauss on Geostationary Transfer Orbit

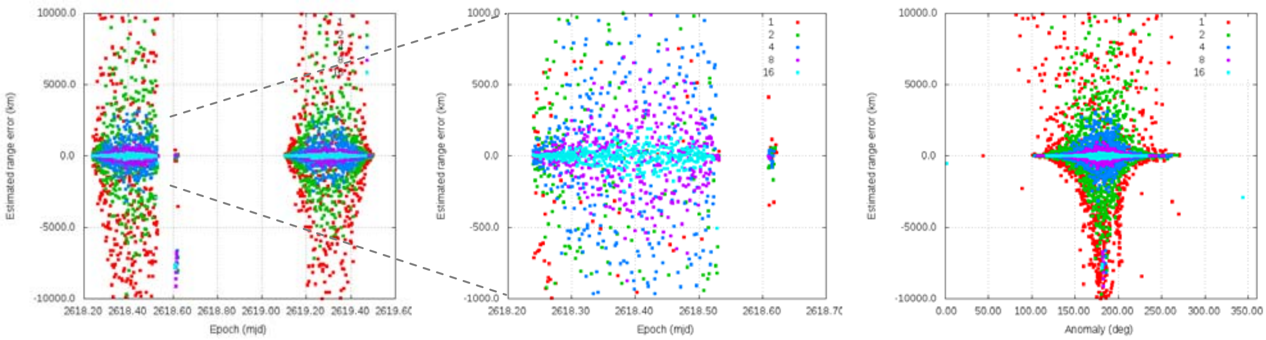

Figure 8. Gooding on Geostationary Transfer Orbit

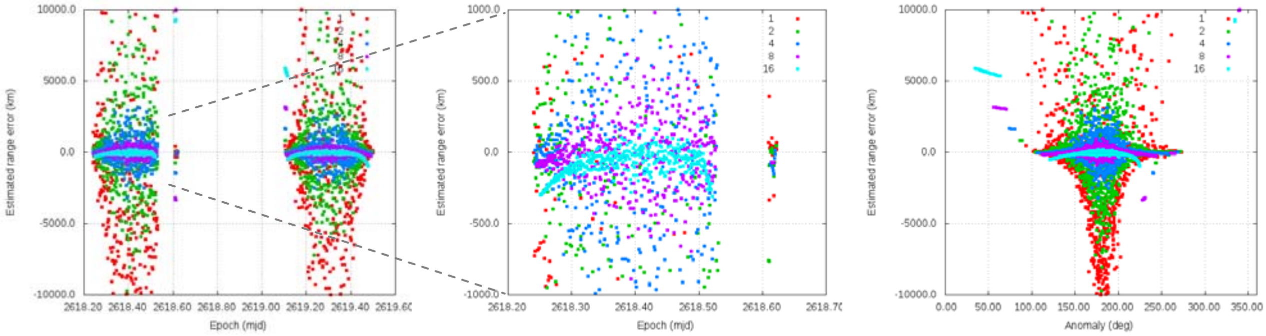

Figure 9. Baker-Jacoby on Geostationary Transfer Orbit

Figure 10. shows the detail on the relative performance of the three algorithms for a GTO. The represented data is for 16 minutes of observation separation. In this figure it becomes obvious that Gooding provides better overall performance although the point to point dispersion is similar in all cases.

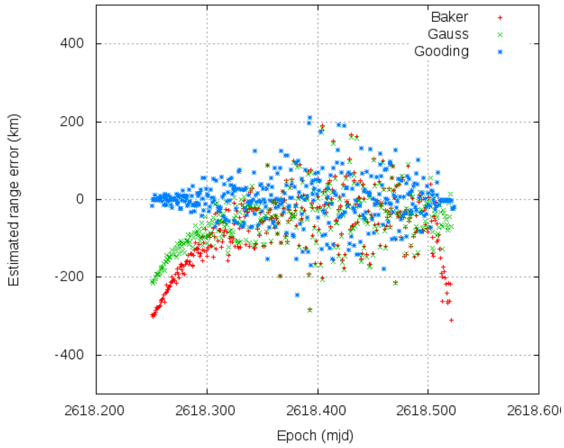

Figure 10. Algorithm performance for GTO

7.2.2 Molniya

Similar analysis as for GTO is performed for the Moniya orbit. In this case only the solution with Gooding is presented in Fig. 11. (performance of Gauss and Baker-Jacoby is slightly worse as in the GTO case). Also like in the GTO case, the performance is worse around apogee and better in the coverage period close to apogee. Illumination conditions will also limit the available time of observation.

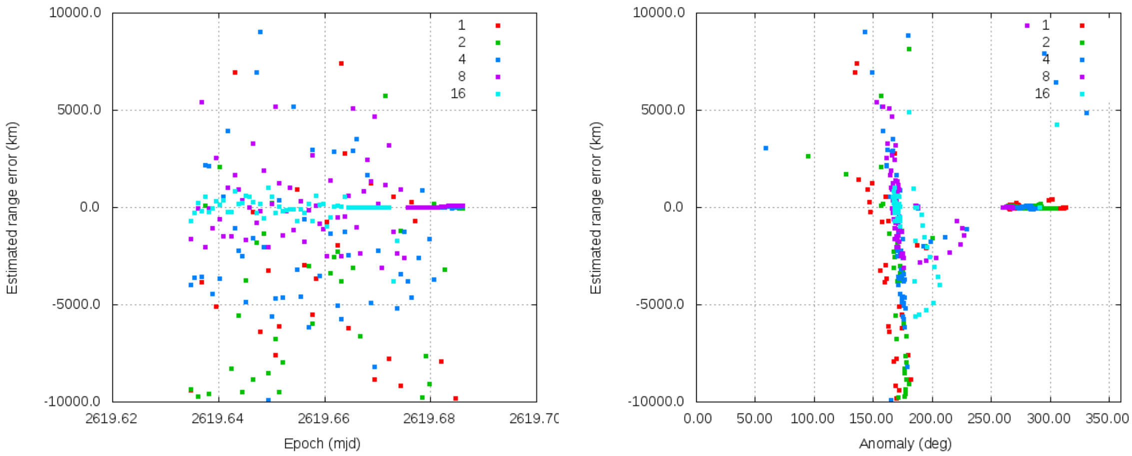

Figure 11. Gooding for Molniya

Figure 12. presents the comparison of performance of the three algorithms where again Gooding provides better performance. It is to be noted that that observations for Gauss and Baker-Jacoby on the left hand side of the plot are out of the scale; Gooding still provides acceptable results there.

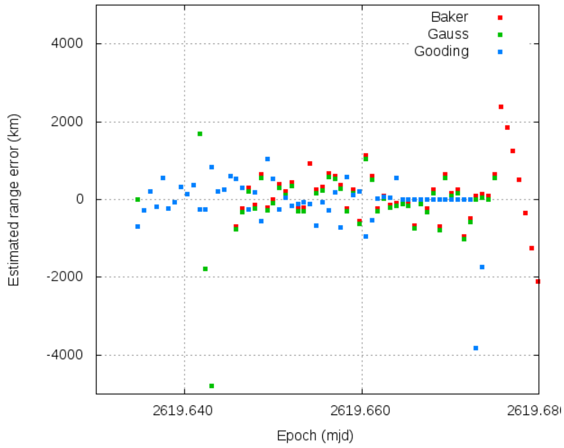

Figure 12. Algorithm performance for Molniya

7.3 Mean Earth Orbits (MEO)

The following figures show two passes of a MEO orbit at 32000 km of semimajor axis for measurement separations between 1 and 16 minutes for the three analysed algorithms. All three algorithms have difficulties in estimating the orbit around the coplanar singularity that appears as a light blue spike on the left hand side plots. Performance outside the singularity is comparable for the three algorithms. In the singularity Gauss performs worse than the other two (points are outside the plot to maintain comparable scales). Although Baker-Jacoby was expected to provide better performance using the fourth observations it is actually Gooding the one that computes better results around the co-planarity.

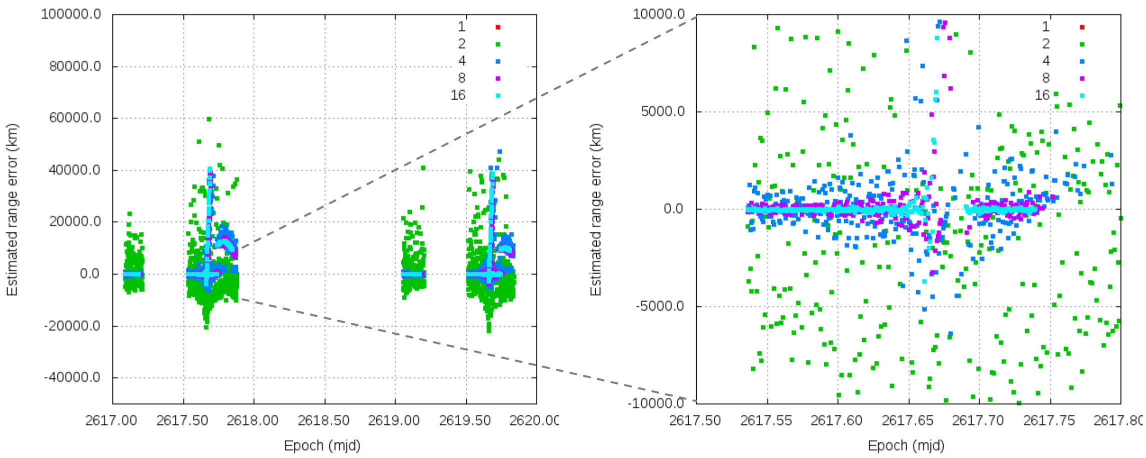

Figure 13. Gauss on MEO at 32000 km

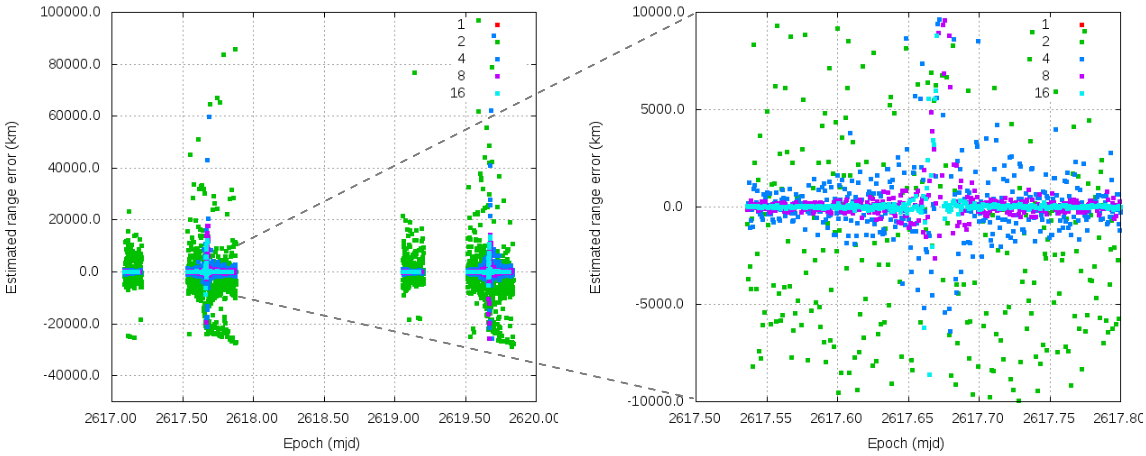

Figure 14. Gooding on MEO at 32000 km

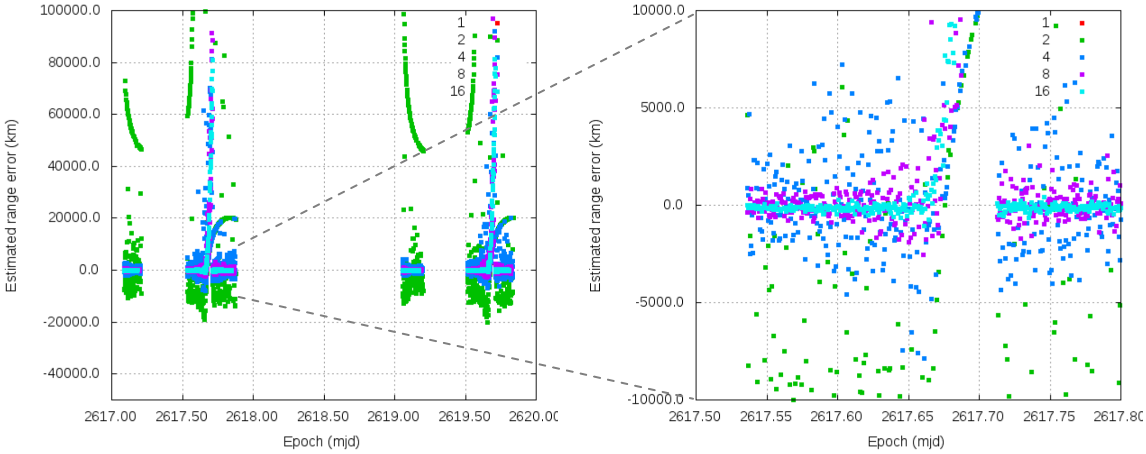

Figure 15. Baker-Jacoby on MEO at 32000 km

The same analysis has been performed for the other two MEO objects at 22000 km and 12000 km of semimajor axis. The relative behaviour of the three methods is similar as seen in the 32000 km case. The following figures illustrate the behaviour for the Gooding algorithm only. The singularity also appears in these cases being Gooding more robust that the other algorithms; in the 12000 km case there are several passes that do not show co-planarity. The level of dispersion of the solutions point to point also decrease with the orbital height as one could expect from the increasing dynamics in the solution.

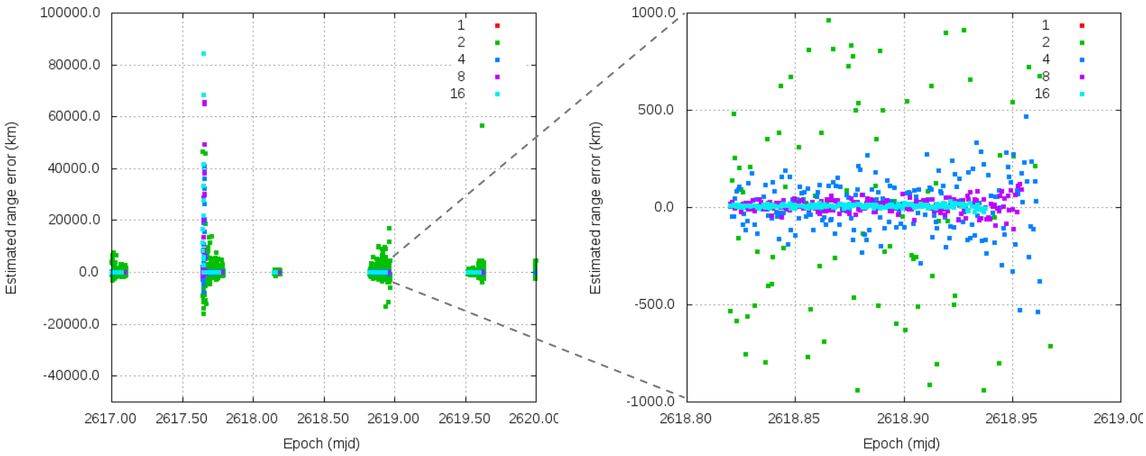

Figure 16. Gooding for MEO at 22000 km

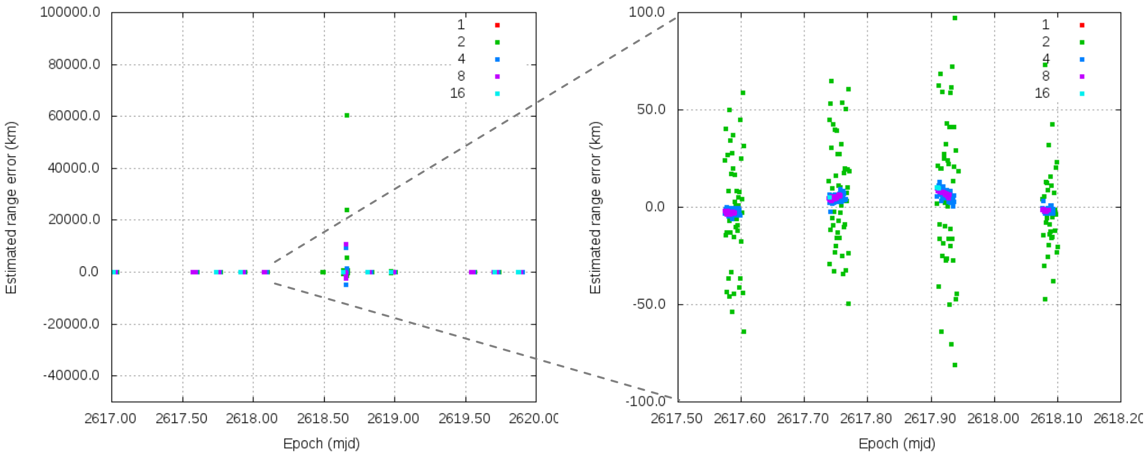

Figure 17. Gooding for MEO at 12000 km

7.4 Low Earth Orbits (LEO)

The LEO case is analysed for two types of orbits: Sun-synchronous and medium inclined altimetry orbit. In this case Baker-Jacoby has been found not to perform adequately and has been removed from the analysis. The level of dispersion in these cases is much lower than for higher orbits due to the presence of the varying geometry (strong dynamics) in the solution. It is however more difficult to fit the more detailed dynamics with the algorithms simplified models and this appears in Fig. 18 and Fig. 19 as there are not zero average solutions for each of the passes. Gooding and Gauss present different levels of offset in this respect but the global behaviour can be considered comparable and the dispersions equivalent. Figures 18. and 19. show data with separations of 2 minutes; the short passes in these orbital scenarios limit the suitable separations to a maximum of 4 minutes; in a real scenario the observational periods are very much reduced due to very limited illumination constraints.

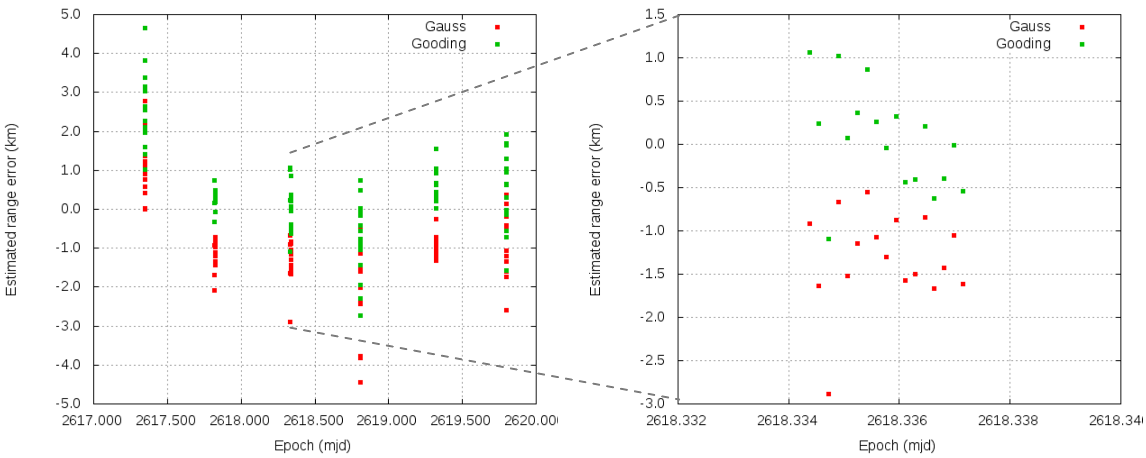

Figure 18. Algorithm performance for LEO (SSO)

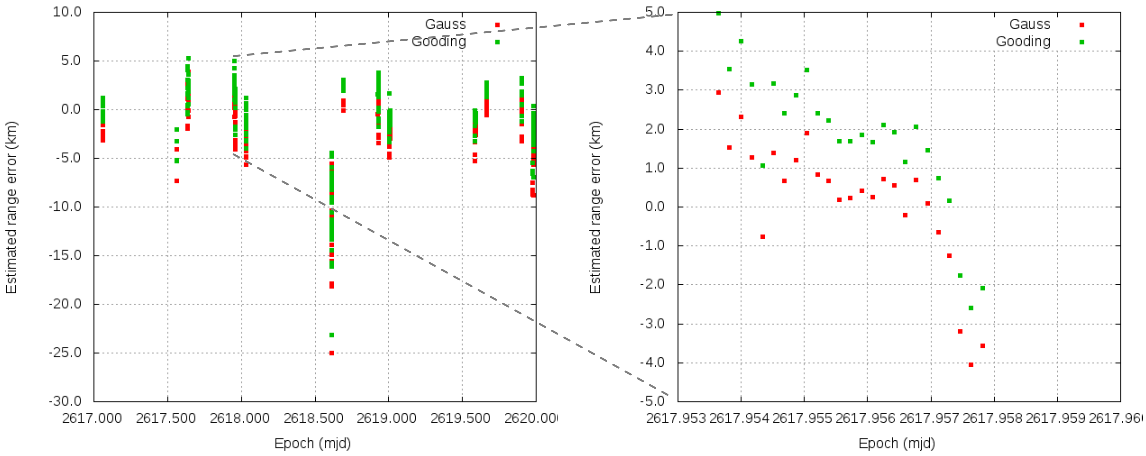

Figure 19. Algorithm performance for LEO (altimetry)

8. Latitude dependency

There is still a missing verification for the potential dependency with the latitude of the observing site. Figure 20. contains the comparison of the solutions computed from the two selected observing sites (continental Spain and the Canary Islands which are separated 13º in latitude); the performance seems to be essentially independent of the latitude.

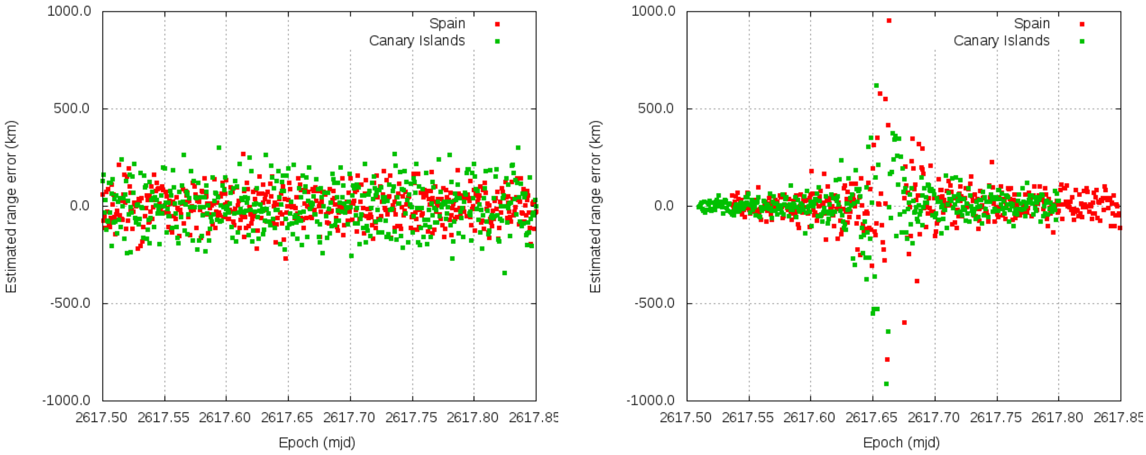

Figure 20. Latitude dependency

9. Conclusion

Analysis of three commonly used algorithms (Gauss, Baker-Jacoby and Gooding) has been performed on a wide variety of orbital scenarios.
The implementation of the algorithms is tested with controlled noiseless simulations that reproduce the original orbit with minimum error. This verification also provides a limit for the maximum attainable accuracy with each of the algorithms.
The first finding (expected) is the weakness of the process of estimating an orbit when just angular measurements are available. Even for low levels of noise (0.0001º in right ascension and declination) the computed solutions present a high level of dispersion for small separations between consecutively taken observations.
In the limiting case when a minimum number of observations is available it is necessary to increase the observation time separation to stabilise the solutions: 16 minutes for GEO, GTO, Molniya and MEO and the maximum possible with the observation limitations for LEO. Without any a-priory assumptions on the type of orbit (e.g. circularity) it is still necessary to collect as many observations as possible to stabilise the solution, then they can be sampled at maximum separations to reduce the dispersion.

The GEO scenario is well characterised and shows the loose behaviour associated to the lack of geometrical variation during the observation period. Here it would be easier to increase the separation since the illumination conditions could last sufficiently.
For MEO objects the coplanar singularity imposes restrictions on the performance. The improvement by Baker-Jacoby that uses a fourth observation to remove the singularity is limited and below expectations.
The process applied to LEO provides better results due to the highly varying geometry although one could expect limitations from the observability opportunities. The shown results may result optimistic depending on the actual object observability.
As a general statement the Gooding algorithm performs better than the rest although there are specific situations in which Gauss and/or Baker-Jacoby produce better results.

10. References

[1] Vallado, D. “Evaluating Gooding angles-only Orbit Determination of Space Based Space Surveillance Measurements”

[2] Escobal, P.R. “Methods of Orbit Determination”

[3] Vallado, D. “Fundamentals of Astrodynamics and Applications”

[4] Gooding, R.H. “A new procedure for orbit determination based on three lines of sight (angles only) ”, April 13, 1993

[5] Baker, R. M. L., Jacoby N.R. Jr. “Preliminary Orbit Determination Method Having no Coplanar Singularity”, July 23, 1975

[6] Milani, A. et al. “Topocentric Orbit Determination: Algorithms for the Next Generation Surveys ”, June 29, 2007