MINIMIZING TOTAL RADIATION FLUENCE DURING TIME-CONSTRAINED ELECTRIC ORBIT-RAISING

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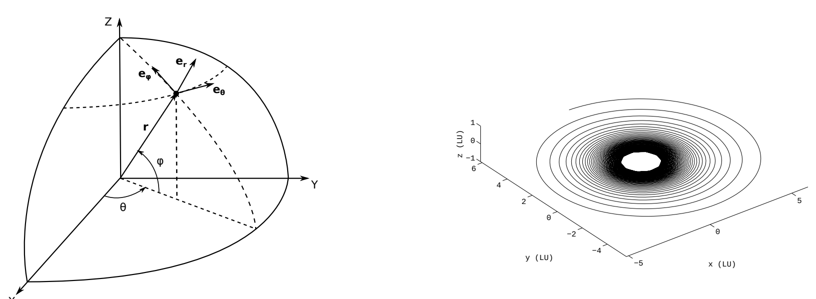
ABSTRACT

The ability to perform orbit-raising to Geosynchronous Earth Orbit (GEO) using electric propulsion (EP) is a key requirement for all future all-electric spacecraft. Long orbit-raising time for a satellite employing an EP device poses a challenge because of the significant high transit time through the Van Allen belt. Incidence of radiation within the belt causes degradation of satellite solar arrays, thereby reducing the available power.[1,2,3] Not only the Beginning-of-Lifetime (BOL) power of the satellite is reduced significantly, the available thrust from the EP device also reduces during orbit-raising and during in-orbit operations. Hence, it is important to restrict the radiation exposure within the Van Allen belt. A number of studies have looked at the trade-offs among transfer time, mass savings and radiation damage for a variety of mission scenarios for electric orbit-raising.[4,5,6,7,8] While some studies have considered electric orbit-raising as only a final phase of orbit-raising maneuver,[6,9] others have considered all-electric orbit-raising and have brought out the importance of launching a satellite to a high-latitude orbit[10] or to Geosynchronous Transfer Orbit (GTO) or a Middle Earth Orbit (MEO).[11] However, all these studies have considered minimum time trajectories through the Van Allen belt.[5,7,10,8,12,11,13] A minimum-time trajectory would correspond to minimum solar cell damage only if the intensity of the radiation flux is uniform throughout the Van Allen belt. In this paper, we seek new orbit-raising trajectories capable of trading some of the time and mass savings for traversing the low-intensity regions of the Van Allen belt, thereby keeping the radiation damage to minimum. To this end, we formulate an optimization problem to minimize the radiation fluence experienced by satellite during passage through the Van Allen belt. We also impose an upper bound on the total orbit-raising time in order to avoid impractically long transfers.



(a) Spherical Reference System

(b) Minimum-time LEO-GEO Transfer

Figure 1. Low-Thrust Optimization.

We describe the motion of a satellite using spherical reference frame, in which (r, θ, ϕ) denotes the location of the satellite (see Figure 1(a)), (u, v, w) denotes the components of velocity and

$\left(T_r, T_\theta, T_\phi\right)$ denotes the components of the thrust provided by the electric engine of the satellite. Considering m to be the mass of the satellite and $T = \sqrt{T_r^2 + T_\theta^2 + T_\phi^2}$, the equations of motion can be written as:

$$\dot{r} = u,\ \dot{\theta} = \frac{v}{r\cos\phi},\ \dot{\phi} = \frac{w}{r},\ \dot{m} = -\frac{T}{c}, \tag{1}$$

$$\dot{u} = \frac{v^2 + w^2}{r} - \frac{\mu}{r^2} + \frac{T_r}{m},\ \dot{v} = \frac{-uv + vw\tan\phi}{r} + \frac{T_\theta}{m},\ \dot{w} = -\frac{uw + v^2\tan\phi}{r} + \frac{T_\phi}{m}. \tag{2}$$

The equations of motion provide the dynamic constraints for our optimization problem. The boundary conditions are given by the initial and final orbits of the satellite:

$$r(0) = r_0^*, \theta(0) = \theta_0^*, \phi(0) = \phi_0^*, u(0) = u_0^*, v(0) = v_0^*, w(0) = w_0^*, m(0) = m_0^*, r(t_f) = r_f^*, v(t_f) = v_f^*. \tag{3}$$

The thrust is bounded by the maximum thrust and thereby the power available for the satellite, while the total orbit-raising time is bounded. In other words, we have

$$0 \le T \le T_{\max}(P),\ t_f \le t_{f,\max}. \tag{4}$$

Note that the available power P during orbit-raising decreases as the solar array degrades during the satellite's transit through the radiation belt. Finally, we would like to minimize the total radiation fluence Ψ (integral of radiation flux Φ over time) that the satellite experiences during its transfer to GEO. In other words, we like to solve the following optimization problem:

$$\min \Psi = \int_0^{t_f} \Phi(r, \theta, \phi)\, dt \text{ subject to set of constraints (1)–(4).} \tag{5}$$

We use a direct optimization methodology to solve the optimization problem described in (5). The methodology converts the problem to a parametric non-linear programming (NLP) problem by discretizing time, state and control variables and setting up algebraic equations to ensure validity of equations of motion at discretized time segments. The NLP is solved by a solver like IPOPT[14] and LOQO. Figure 1(b) shows a coplanar ($\phi = 0$) minimum-time trajectory from Low-Earth Orbit (LEO) to GEO computed using the direct optimization methodology. The trajectory does not incorporate any out-of-plane thrusting as it minimizes time but passes through the entire high intensity regimes of the Van Allen belt, thereby incurring maximum radiation damage. If the satellite could perform plane change (radiation intensity is less at higher latitude ϕ), it could potentially decrease some of this damage at the expense of higher orbit-raising time and lower mass savings. The formulation presented in the current paper is capable of determining trajectories that aim to incur minimum radiation damage and to keep the total transfer time within reasonable limit. In the full version of the paper, we discuss our formulation in greater detail. We consider that the satellite starts orbit-raising from an orbit outside the Van Allen belt (LEO, Sun-synchronous Polar Orbit) or from an orbit that is partially (GTO) or completely (MEO) within the Van Allen belt. We illustrate via numerical examples how the minimum radiation trajectories differ from minimum-time trajectories. We also demonstrate the impact of the transfer time upper bound on the optimal solution. Finally, we present the trade-offs among transfer time, mass savings and radiation fluence for different choice of injection orbits and different EP devices.

1. References

[1] Smits, F., “The Degradation of Solar Cells under Van Allen Radiation,” *IEEE Transactions on Nuclear Science*, Vol. 10, No. 1, 1963, pp. 88–96.

[2] Flanagan, P. F., Horsewood, J. L., and Pines, S., “Precision Orbit Raising Trajectories,” *AIAA 11th Electric Propulsion Conference*, New Orleans, LA, Mar 19-21 1975.

[3] Dailey, C. and Lovberg, R., “Shuttle to GEO Propulsion Trade Offs,” *18th Joint Propulsion Conference*, Cleveland, OH, Jun 21-23 1982.

[4] Redding, D., “Avoiding the Van Allen Belt in Low Thrust Transfer To Geosynchronous Orbit,” *AIAA 21st Aerospace Sciences Meeting*, Reno, NV, Jan 10-13 1983.

[5] Ennix, K. A., Dickey, M. R., Klucz, R. S., and Matuszak, L. W., “Electric vehicle Analyszer,” *21st International Electric Propulsion Conference*, Orlando, FL, July 18-20 1990.

[6] Free, B., “High Altitude Orbit Raising with On-Board Electric Power,” *International Electric Propulsion Conference*, Seattle, WA, Sep 1993.

[7] Fitzgerald, A., “The Effect of Solar Array Degradation in Orbit-Raising with Electric Propulsion,” *International Electric Propulsion Conference*, Seattle, WA, Sep 1993.

[8] Biagioni, L., “Evolutionary Optimization of Launch Vehicle Electric Propulsion Integration for GEO Missions,” *Joint Propulsion Conference*, Huntsville, AL, July 2000.

[9] Kim, K., *2010 A Novel Numerical Analysis of Hall Effect Thruster and its Application in Simultaneous Design of Thruster and Optimal Low-Thrust Trajectory*, Ph.D. thesis, Georgia Institute of Technology, July 2010.

[10] Oleson, S. R. and Myers, R. M., “Launch vehicle and power level impacts on electric GEO insertion,” *Joint Propulsion Conference and Exhibit*, Buena Vista, FL, July 1-3 1996.

[11] Duchemin, O. B., Caratge, A., Cornu, N., Sannino, J.-M., Lassoudire, F., and Lorand, A., “Ariane 5-ME and Electric Propulsion: GEO Insertion Options,” *47th Joint Propulsion Conference and Exhibit*, 2011.

[12] Byers, D. C. and Dankanich, J. W., “Geosynchronous-Earth-Orbit Communication Satellite Deliveries with Integrated Electric Propulsion,” *Journal of Propulsion and Power*, Vol. 24, No. 6, Nov-Dec 2008, pp. 1369–1375.

[13] Dutta, A., Libraro, P., Kasdin, N. J., and Choueiri, E., “A Direct Optimization Based Tool to Determine Orbit-Raising Trajectories to GEO for All-Electric Telecommunication Satellites,” *AAS Astronautical Specialist Conference*, Minneapolis, MN, Aug 2012.

[14] Wchter, A. and Biegler, L. T., “On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming,” *Mathematical Programming*, Vol. 106, No. 1, 2006, pp. 25–57.