NATURAL RECOVERY OPTIONS FOR FAILED TRANSFERS TO THE SUN-EARTH LIBRATION POINTS WITH THE EXAMPLE OF GAIA

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Abstract: *The libration points of the Sun-Earth system are attractive destinations for astrophysics missions that require a stable thermal environment and rigid orbit geometry. In the recent past those missions have been implemented as observatory missions on large-amplitude quasi-halo orbits, that also allow a free transfer from injection by the launcher onwards, or as scanning missions on small-amplitude Lissajous orbits that require an insertion manoeuvre onto the stable manifold of the target orbit. All those missions exploit the benign environmental conditions and low transfer cost of this mission option. Looking at the concrete mission example of the scanning mission Gaia, we find that the transfers to those target orbits provide another advantage: a low cost recovery option from a failed transfer. The stable manifold of small libration orbits intersects with the Moons orbit at a lunar phase of approximately* $300°$ *with respect to the full Moon. If the injection by the launcher fails in the typical way of providing too low an apogee (typical target values to reach the libration point range from* 0.9×10^6 *to* $1.5 \times 10^6\,\mathrm{km}$ *depending on transfer strategy), the spacecraft will be stranded on a highly elliptical Earth orbit with an inertially fixed apse line roughly pointed towards the anti-Sun direction (i.e.* $360°$ *lunar phase, full Moon). Since the Sun-Earth line moves at* $1°$ *per day, the apse line will be at* $300°$ *Moon phase after two months. If maneuvering capability on board the spacecraft is retained, careful phasing to reach apogee at this position will allow a gravity assist manoeuvre to jump onto the stable manifold of the target orbit with a low cost in* Δv.

Keywords: *mission design, three-body-problem, Gaia, libration point, contingency*

1. Introduction

Robert Farquhar [1] recognised in the 1960ies that the libration points of equilibrium of the restricted circular three-body problem are potentially interesting targets for space missions. While the first proposed application was for lunar data relay purposes in the Earth-Moon system, the first real mission (ISEE-3) occurred in the late 1970ies to the day-side near-Earth libration point L_1 of the Sun-Earth system [2]. The dynamics at the libration point have been described in exhaustive detail in the literature [3, 4, 2, 5, 6, 7, 8]. The mission analysis section of the European Space Agency has so far performed detailed mission analysis for four missions to the libration points of the Sun-Earth system: SoHO, Herschel and Planck, and most recently Gaia. Orbits around L_2 fall in two categories depending on the mission profile: sky survey missions that utilise a spinning platform and observatory missions that require a very stable inertial three-axis pointing. These two spacecraft types, together with the desire to avoid complex mechanisms on board, drive the orbit selection in two directions, because the solar arrays (and Sun shields) have to be pointed at the Sun and any directed antenna at the Earth. On an orbit around L_2 the Earth appears to be moving around the Sun as the spacecraft liberates about the libration point. The angular separation of Sun and Earth as well as its variation depends on the *Lissajous elements* of the Libration motion, that is on the periodic in- and out-of plane amplitudes and the in- and out-of plane phases [9, 10].

Requirements on the geometry of the spacecraft spin axis coming from survey missions like Gaia strongly constrain the options for the configuration of the on-board antenna, putting a requirement on the maximum range in the angle formed by the triangle Sun-spacecraft-Earth $\theta_{\max}$. Because the phased-array antenna of Gaia is capable of covering a variation of the Sun-spacecraft-Earth angle

For the Gaia mission a Soyuz launcher was selected with a Fregat upper stage, which was the same combination as was chosen for the ESA Cluster II missions (the Fregat was upgraded after the Cluster II mission and the upgrade was used for Gaia). A risk mitigation measure for a severe launcher underperformance was due, because there was an issue with the Soyuz during the Cluster launch. Taken from an operational report of the Cluster mission: "Around 20 minutes after launch it was reported that an underperformance of the Soyuz launcher of about $100\,\mathrm{m\,s^{-1}}$ occurred and this was compensated by the Fregat first burn. As a consequence, it was anticipated that the Fregat second burn could be about 30 to $70\,\mathrm{m\,s^{-1}}$ less than nominal, ... The underperformance could be recovered by the spacecraft and lead to a slightly higher propellant usage and a different fuel/oxidiser remaining on spacecraft 1 and 4 at end-of-mission. Due to the near-parabolic injection of Gaia, the consequences of a similar underperformance would be more severe.

Should a severe launcher underperformance leave the Gaia spacecraft on an orbit with insufficient energy to reach the libration orbit beyond the maneuvering capabilities of the spacecraft, the only way to recover from that situation is to exploit a lunar gravity assist to increase the orbital energy to the required level. In order for this strategy to be successful the gravity assist manoeuvre must occur when the Moon passes through the stable manifold that is associated with the Lissajous orbit (Sun-spacecraft Earth angle constraint to $\theta_{\max} \leq 15^\circ$ in the case of Gaia, see figure 1 in an accompanying paper [11]). In position this occurs at the 300° lunar phase, i.e. at two third waxing Moon (the stable manifold is a two-dimensional tube, which is invariant in the Sun-Earth rotating frame, in which the azimuth of the Moon is approximately equal to the lunar phase with 0° representing the full Moon). Because of the large semi-major axis the apse line of the initial orbit is inertially fixed. In the rotating frame of the Earth and Sun this means that the orbit's apse line rotates by -360° in a tropical year. The time it takes the apse line to reach the 300° position from its initial 15° is thus 75 d. The lunar fly-by will put the spacecraft on the manifold of the intended target orbit, so that the large Δv that is allocated in the nominal mission to the insertion manoeuvre (small amplitude Lissajous orbits cannot be reached directly from a launcher injection due to the spatial separation of their associated stable manifold from the Earth) can be used for the required phasing, stabilisation, and plane change manoeuvres in Earth orbit.

2. Space and Ground Segment Constraints of Gaia

The space segment of the Gaia mission comprises a dedicated bus with integrated payload and service modules. While the payload module is a fascinating piece of high precision astronomy equipment, it is the service module that determines the maneuverability and the capability of the spacecraft to cope with a contingency situation. The Gaia service module is equipped with 8 hydrazine thrusters (actually 16 as all are individually double-redundant) each delivering a thrust of 10 N. The small cold gas thrusters that are used for the attitude control in the target orbit are not considered here due to their limited total Δv capability. The layout of the hydrazine thrusters is shown in figure 1.

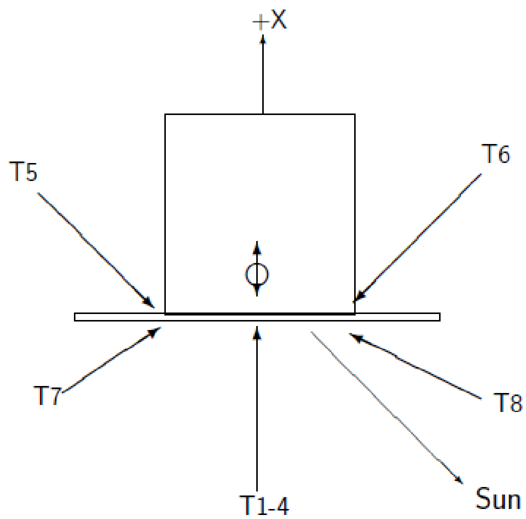

Figure 1. Configuration of the hydrazine thrusters on the Gaia spacecraft ($+x$-axis defined as perpendicular to the Sun shield). The large central box represents the thermal tent containing the payload module and the horizontal bar below it represents the Sun shield (no to scale). The arrows indicate the direction of acceleration provided by each thruster (T). The circle-and-double-arrow in the middle of the thermal tent indicates the average position of the centre of mass and its motion due to fuel depletion.

The thruster configuration is such that large manoeuvres have to be performed by thrusters 1 to 4, which together generate a thrust along the $+x$ direction. Thrusters 5 and 6 cannot be used for other than very small manoeuvres ($< 1\,\mathrm{m\,s^{-1}}$) due to limitations of the fuel management system. It is assumed below that ballistic parts of the trajectory are flown in a fixed attitude relative to the Sun and manoeuvres are performed by slewing the spacecraft $+x$ axis into the manoeuvre direction. Other subsystems of the Gaia spacecraft are the solar power generators, the thermal control, and the telemetry and tele-commanding systems. While the constraints imposed by the limitations of these systems have not been analysed in detail for the recovery operations, it is assumed that in a contingency situation some of the mission rules for the nominal missions can be waived.

In terms of Δv it is assumed here that the allocation for the first trajectory correction manoeuvre combined with the orbit insertion manoeuvre is available for the recovery operations. None of those two manoeuvres are required for the recovery. The amount of geometric Δv allocated to those two manoeuvres is $235\,\mathrm{m\,s^{-1}}$ [11].

Not many restrictions in the performance of the ground segment are considered, as normally in a severe contingency situation many resources on the ground can be made available. The only restriction under consideration is that the first manoeuvre may be performed only 48 h after launch if at all possible in order to allow sufficient time for tracking and orbit prediction.

3. Construction of Recovery Mission from Severe Launcher Underperformance

We assume that a failed launcher injection results in an highly elliptical orbit with all but one orbit parameters the same as the baseline target orbit. For the actual launch date of Dec 19th, 2013, we have the possible injection states given in table 1.

parameter	value	unit
injection epoch	19-Dec-2013 09:55:24 UTC	
perigee radius	6,722.288	km
apogee radius	188,000 to 780,000	km
inclination	15°	
right ascension of ascending node	14.8°	
argument of perigee	265.5°	
true anomaly	52.3°	

Table 1. Orbital parameters of the initial injection state.

The orientation of the line of apsides for that orbit is such that it would lead to a fast transfer towards the target libration orbit if the orbital energy was sufficient – it is closely aligned with the Sun-Earth line. More precisely, the minimum Δv transfers towards a small amplitude Lissajous orbit can be found if the transfer apse line has an angle of $\approx +15^\circ$ with respect to the Sun-Earth line. As a consequence the orientation of the nominal transfer is such that an apogee of a highly elliptical orbit associated with a launcher underperformance contingency is naturally oriented such that the apogee is in the first quarter of the rotating frame (with the $+x$-axis pointing parallel to the Sun-Earth line), where the solar gravity gradient is pointing in the retrograde direction. As a consequence, if the reached apogee is still very high (above the Moon's orbit) the first action to be taken in the case of a launcher under performance is the stabilisation of the perigee, which would otherwise be lowered to below the Earth surface, causing a reentry into the Earth's atmosphere if no action was taken. This is an example of a set of general rules that have been applied in the construction of the recovery strategies:

- Constrain perigee radii to the interval $[6531\,\mathrm{km}; 10,000\,\mathrm{km}]$ by manoeuvres at large Earth distances
- For large orbits, reduce the semi-major axis in order to allow multiple revolutions before lunar fly-by 75 d after the launch
- The orbital motion of the spacecraft is phased such that the one or two encounters of the Moon with the apse line should occur when the spacecraft is at perigee in order to avoid strong lunar perturbation before the actual fly-by
- No deterministic manoeuvre at the last perigee prior to the lunar fly-by in order to avoid trajectory correction on the short arc between Earth and Moon
- Combine manoeuvres where possible (e.g. apogee lowering and inclination change)

Starting from the nominal injection state we consider three recovery strategies (in the order of decreasing apogee radius of the reached orbit): the 3, 5, and 6 manoeuvre sequence. The intention in this approach is to have a feasible solution in the situation when the contingency occurs, which could then be refined and adjusted to the situation at hand. In this sense the three strategies are not optimal, as they serve templates for more dedicated recovery strategies. These strategy have one advantage, they can be applied to a whole range of apogee radii (that is orbital energy), so that the required envelope in Δv and manoeuvre timing can be explored. The range in apogee radii that needs to be covered by the strategies covers the low end, at which the orbital energy is insufficient to reach the lunar orbit ($\approx 150,000\,\mathrm{km}$) and the high end ($\approx 770,000\,\mathrm{km}$), above which the Gaia spacecraft is capable to increase the orbital energy directly with a correction burn 48 h after launch. If the launcher leaves the spacecraft stranded with an apogee altitude in this range, the orientation of the apse line of the resulting highly elliptical orbit (HEO) is almost inertially fixed. This means, in the rotating frame of the libration orbit, in which the Sun-Earth line is identical with the positive x-axis, the HEO rotates by approximately 1° per day due to the rotation of the reference frame around the z-axis. The apse line reaches therefore naturally a position of -60° relative to the Sun-Earth line, which coincides with the intersection of the stable manifold associated to the target libration orbit with the orbit of the Moon.

A lunar gravity assist manoeuvre can therefore be utilised to jump onto the target stable manifold without the need for an insertion maneovure, provided the asymptotic velocity vector of the fly-by can be made to match the local velocity of the stable manifold. For this to happen two conditions must be met: neither the inclination nor the deflection angle of the fly-by hyperbola must not bee too low (high deflection angle applies low hyperbolic eccentricity). This saving in transfer Δv will partially be used to stabilise the HEO during the two months with multiple revolutions around the Earth against luni-solar perturbations, to enable the phasing to target the perigee for the lunar gravity assist arc, and to perform trajectory correction manoeuvres during the transfer. Here we analyse only the reference transfer, no navigation analysis is performed. The resulting transfer trajectories are plotted in figures 2, 3, and 4 for the 3, 5, and 6 manoeuvre strategy, respectively.

Here we discuss the tactics to construct the 3 manoeuvre strategy for apogees above the Moon, but below $770,000\,\mathrm{km}$. At very high apogee radii the orbital period is sufficiently long so that the time it takes the spacecraft to reach apogee the apogee lies in the fourth quarter of the rotating Sun-Earth frame, effectively raising the perigee, so that no immediate perigee raising manoeuvre is required. The next perigee will be reached after approximately 28 d, which is close enough to a sidereal month, so that a manoeuvre past apogee can be used to lower the apogee to reach the perigee earlier, i.e. at the time when the Moon crosses the apse line the first time. This tactic avoids instability form the first lunar encounter. Due to the apse line being almost at 90° with respect to the nodal line it is however not possible to adjust the inclination of the orbit to the value of $\approx 30^\circ$ that is required for the arc prior to the fly-by in order to reach the right velocity direction for the stable manifold. This task is kept for the encounter with the nodal line just before the first perigee. In an effort to combine inclination change and apogee lowering, the second manoeuvre takes place at true anomalies between -90° and 0°. The apogee lowering is required in order to allow a sufficient deflection angle δ for the flyby. Since in the two-body approximation it is true that

$$\sin\frac{\delta}{2} = (1 + \frac{b^2 v_\infty^4}{\mu^2})^{-\frac{1}{2}},$$

where v_∞ is the asymptotic velocity relative to the Moon, b the impact parameter. Here too high an apogee altitude of the trans-Moon arc leads to large v_∞ and thus to small deflection angle. Therefore, the apogee of the arc before the fly-by must not be significantly above the Moon's orbit. The final manoeuvre of the 3 manoeuvre strategy is located at the second perigee and is used for phasing.

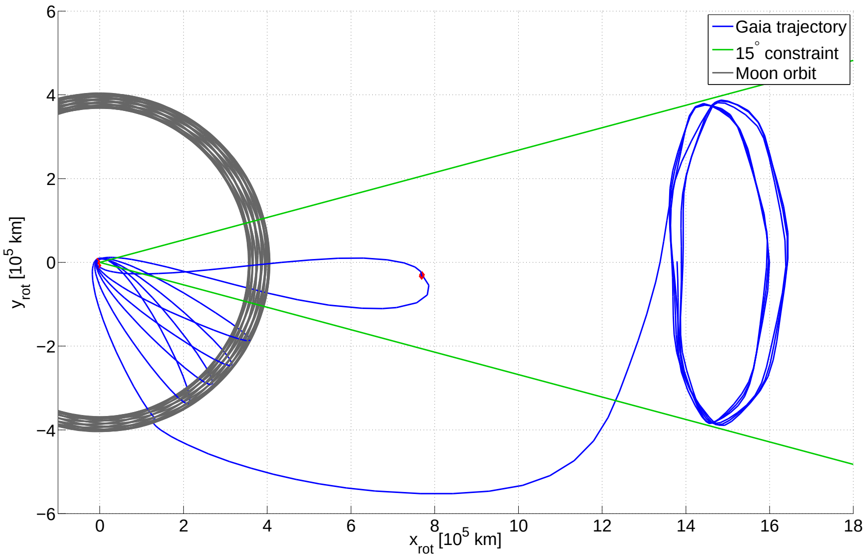

Figure 2. Trajectory of a successful 3 manoeuvre transfer to the target libration orbit after a severe underperformance launch failure resulting in a high apogee of $768,000$ km. The transfer and target trajectories are shown in blue. The gray line represents the orbit of the Moon during the same time frame. The red markers indicate the locations of the 3 manoeuvres.

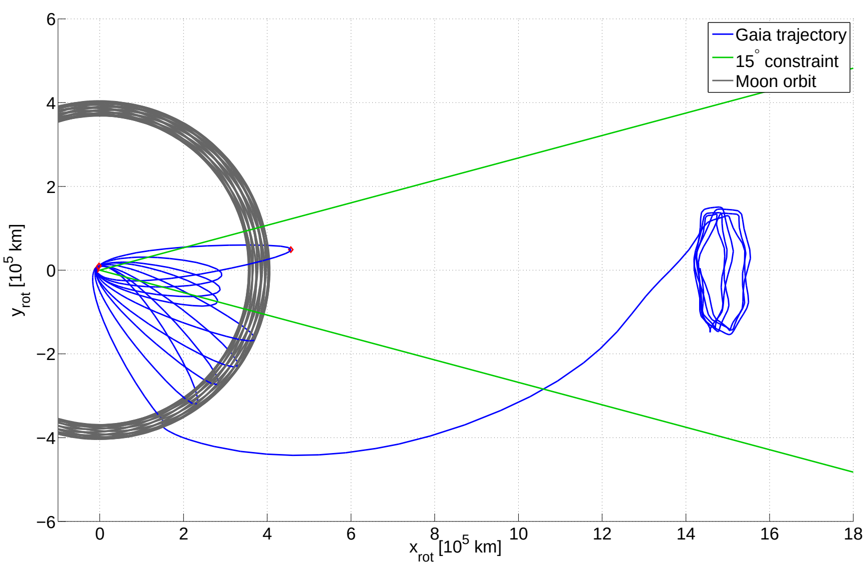

Figure 3. Trajectory of a successful 5 manoeuvre transfer to the target libration orbit after a severe underperformance launch failure resulting in a moderate apogee of $468,000$ km. The transfer and target trajectories are shown in blue. The gray line represents the orbit of the Moon during the same time frame. The red markers indicate the locations of the 5 manoeuvres.

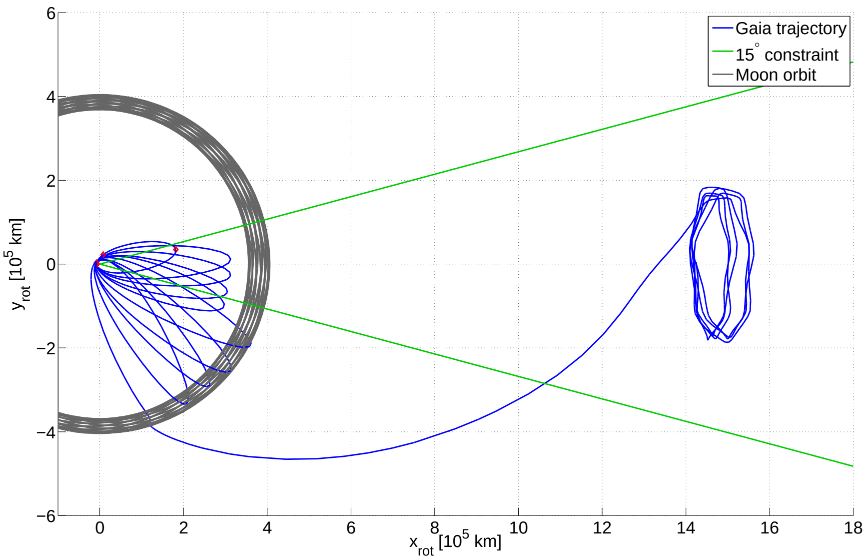

Figure 4. Trajectory of a successful 6 manoeuvre transfer to the target libration orbit after a severe underperformance launch failure resulting in a low apogee of $188,000\,\mathrm{km}$. The transfer and target trajectories are shown in blue. The gray line represents the orbit of the Moon during the same time frame. The red markers indicate the locations of the 6 manoeuvres.

The rest of the transfer (3 revolutions on the highly elliptical orbit leading up to the fly-by) is spent without deterministic manoeuvres.

The strategy must be changed as the reached apogee radii become too small to exploit the loiter period at the first apogee in order to avoid the first Moon encounter: At the first perigee the apogee must be lowered well below the Moon's orbit. This is certainly sub-optimal to some extend, as at the end of the sequence the apogee must be raised again to the Moon's orbit in preparation for the Moon fly-by. Like for the 3 manoeuvre strategy the inclination change manoeuvre is combined with the first perigee maneovure. However, prior to the first apogee lowering manoeuvre there must be a perigee raising manoeuvre at the first apogee, because due to the lower orbital period of the initial orbit the apogee remains in the first quadrant, in which the perigee is lowered due to the Sun's gravity gradient.

At the higher end of initial apogee radii the time to the first apogee is still sufficient to fulfill the minimum time before manoeuvre requirement of $48\,\mathrm{h}$. The third manoeuvre is again at perigee in order to phase for the Moon avoidance manoeuvre, the fourth manoeuvre raises the apogee back to the Moon's orbit after the Moon has passed above the apogee. Finally the last manoeuvre of the 5 manoeuvre strategy provides the phasing towards the lunar fly-by at the first perigee of the final four revolutions prior to the fly-by. It can be seen from figure 3 that the target orbit for this strategy is smaller than the baseline $\theta_{\max} = 15°$. This is acceptable to the Gaia mission as the communication geometry of the spacecraft's phased-array antenna works also for smaller Sun-spacecraft-Earth angles. It should be noted that there could be eclipses of the Sun by the Earth on the smaller Lissajous orbits that have to be mitigated by an avoidance manoeuvre that has normally a size of less than $15.1\,\mathrm{m\,s^{-1}}$.

As the launcher underperformance becomes more and more severe and with it the initial apogee radius smaller and smaller, the strategy must be changed again. While the perigee raising manoeuvre at the first apogee remains, it is now scheduled as early as 24 h after launch. This is unavoidable if a reentry into the Earth's atmosphere is to be avoided as the orbital period becomes smaller than 2 d . Now it is no longer necessary to lower the apogee in order to avoid the first lunar encounter, and so the second manoeuvre is an apogee raising manoeuvre at the first perigee. Again due to the relatively small initial orbit the inclination change manoeuvre (third in the sequence) is not combined with the first manoeuvre at perigee, but is put at the second crossing of the line of nodes on the first raised orbit. The fourth manoeuvre in the sequence adjusts the phasing for the Moon avoidance. It is followed by the fifth manoeuvre, which raises the apogee further after the Moon has passed over the apogee. Finally the last manoeuvre of the 6 manoeuvre strategy raises the apogee back to the lunar orbit in order to prepare the lunar fly-by, which is preceded by four ballistic revolutions.

All the solutions discussed above have the advantage that at the final orbit before the lunar gravity assist there are no deterministic manoeuvres. This is very beneficial for the operational safety of the navigation procedures as it is not required to perform a correction manoeuvre during the Earth-Moon arc.

After construction of the three recovery strategies, we can employ a parametric analysis of the transfer Δv as a function of the initial apogee radius. The result is shown in figure 5. It can be seen that always one of the three recovery strategies can be identified that is below the Δv limit for initial apogee radii above $190,000$. Of course the time to the first manoeuvre is also affected by the initially reached apogee altitude: the associated diagram is shown in figure 6. The operational requirement of having at least 48 h before the first manoeuvre is fulfilled for apogee radii above $210,000\,\mathrm{km}$. It can be expected that in the desperate case of a lower injection the operations team would be ready to sufficiently accelerate the time to first manoeuvre.

While the case analysed above is limited to launches around the actual launch period from Dec 19th to 22nd 2013 (Gaia actually launched on the 19th), the same methodology can be used to find recovery transfer for any launch date. This is because of the natural geometry connecting the baseline launch conditions to the $-60°$ transfer towards a lunar gravity assist inside the stable manifold of the target orbit. The necessary synchrony with the Moon's motion can be generated by simple phasing.

4. Conclusion

We have presented our efforts to increase the operational robustness of missions to the Sun-Earth libration points by preparing recovery strategies utilising a lunar gravity assist technique. Thanks to the natural geometry of the stable manifolds of libration orbits and its intersection with the Moon's orbit there are natural transfers with fly-bys approximately two months after the launch.

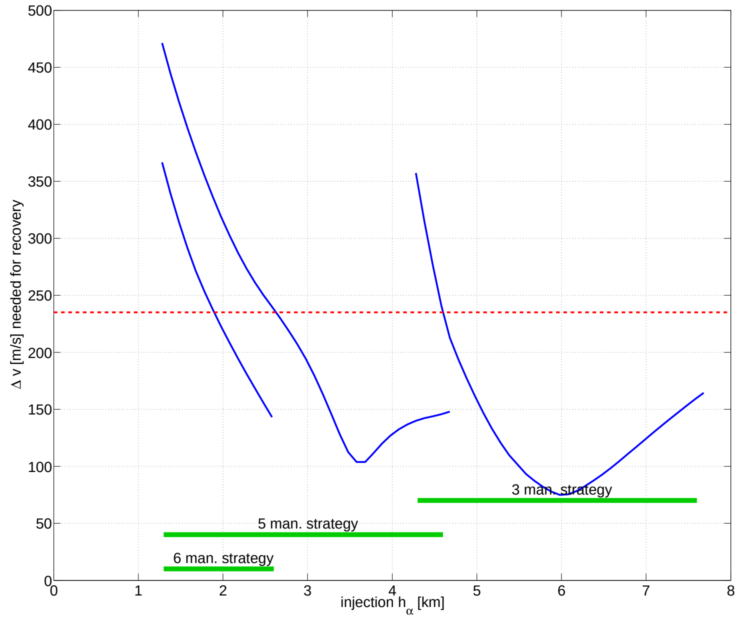

Figure 5. Transfer Δv for the various manoeuvre strategies explained in the text. The dashed line indicates the Δv available for recovery.

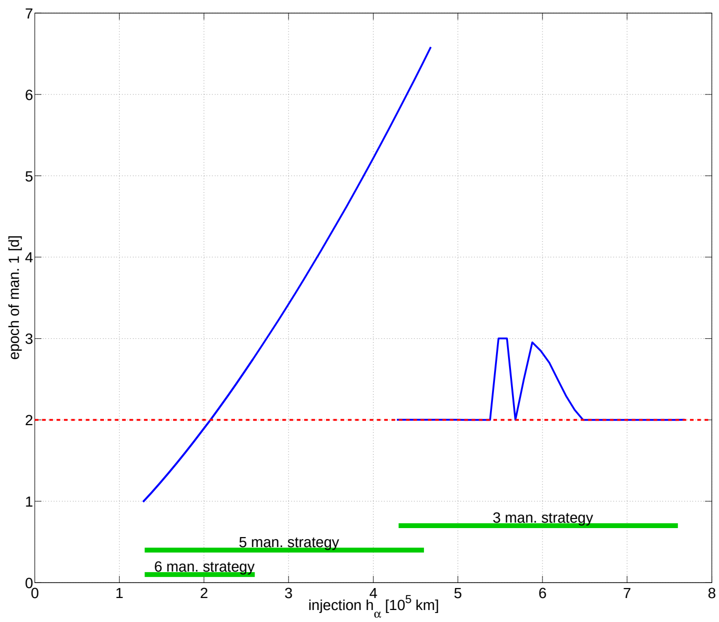

Figure 6. Time to first manoeuvre for the various manoeuvre strategies explained in the text. The dashed red line indicates the constraint of minimum 2 d applied to the 5 and 3 manoeuvre strategies.

The required transfer Δv is spent on perigee stabilisation, apogee raising/lowering and inclination change. Both inclination change and apogee lowering are required in order to allow the outgoing asymptote of the lunar fly-by be aligned with the stable manifold at the intersection point. The perigee stabilisation is needed to avoid either reentry to the Earth's atmosphere for small initial apogees or too high perigees (and thus inefficient follow-on manoeuvres) for high initial apogees. A recovery of the Gaia mission for all apogees above $190,000\,\text{km}$ could be found that are within the mission constraints (except for an early manoeuvre approximately 36 instead of 48 h after launch).

The same method will not work equally well for missions on free insertion large amplitude quasi-halo orbits around the Sun-Earth libration points, because for those missions much lower Δv budgets are the norm. For the small amplitude Lissajous missions like Gaia the question arises whether the mission could not have designed for the lunar gravity assist right from the start in an effort to maximise the final payload mass – after all: some of the recovery missions require sufficiently less Δv than the nominal mission. This is however not recommended from an engineering and risk management point of view, as such a mission comes with a significantly increased operational effort and risk as well as much stronger performance requirements on the space segment. Sophisticated recovery strategies should be employed only in the consolidated phase of a robust nominal mission in order to mitigate some of the operational risks associated with a rocket launch. As a final remark we can report that Gaia successfully launched on Dec 19th, 2013 and has reached its operational orbit around the Sun-Earth libration point on January 14th, 2014.

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