GPS-Based Precise Orbit Determination for LEO Satellites with Carrier-Phase Integer Ambiguity Resolution

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Abstract: *Japan Aerospace Exploration Agency (JAXA) has developed a GPS-based Precise Orbit Determination (POD) software implementing an ambiguity fixing procedure and empirical Phase Center Variations (PCVs) corrections for Low Earth Orbit (LEO) satellites. The software can estimate orbits of LEO satellites with an accuracy of a few centimeters, which meets the requirements of Japanese ocean surface topography mission JAXA has proposed. This paper explains a brief overview of the POD software and accuracy evaluation results of orbits obtained by the software for the Gravity Recovery and Climate Experiment (GRACE) satellite.*

Keywords: *GPS, Orbit Determination, Integer Ambiguity Resolution, PCVs, GRACE.*

1. Introduction

Currently improvements on the accuracy of orbit determination for Low Earth Orbit (LEO) satellites are highly demanded to conduct missions like synthetic aperture radar or ocean surface topography observation. Japan Aerospace Exploration Agency (JAXA) has proposed the first ocean surface topography mission in Japan [1]. In this mission, JAXA will deliver near real time products to users within a few hours as well as precise products which require a radial orbit accuracy of a few centimeters. In order to meet these requirements, JAXA has developed the GPS-based Precise Orbit Determination (POD) software, which can estimate orbits of LEO satellites with an accuracy of a few centimeters.

At the beginning of this paper, a processing strategy and models used in the POD software including the procedure of Integer Ambiguity Resolution (IAR) are summarized. Afterward, the POD results with IAR are evaluated using actual flight data of the Gravity Recovery and Climate Experiment (GRACE) satellite. Finally, the evaluation results using the empirical Phase Center Variations (PCVs) estimated by the POD software are described. These results indicate that the POD software can estimate orbits of LEO satellites with an accuracy of a few centimeters.

2. Software Packages

JAXA developed the Global Navigation Satellite System (GNSS) precise orbit and clock estimation software “MADOCA” in 2011–2012, which can estimate GNSS orbits with an accuracy of a few centimeters [2]. When developing the POD software, the capabilities of MADOCA were expanded to cover not only GNSS but also LEO satellites making use of the observation and dynamical models as well as the parameter estimation algorithm that were already implemented in MADOCA.

This chapter describes the processing strategy and models implemented in the POD software for LEO satellites.

2.1. Processing Strategy

One of the key functions for the precise orbit determination with GPS measurements is integer ambiguity resolution. GPS carrier-phase measurements have the ambiguities which have theoretically integer nature but cannot always be fixed to integers because of fractional-cycle biases (FCBs) in the GPS measurements. For a large network of GPS receivers, integer resolutions of these ambiguities can be routinely performed. To resolve as many integer ambiguities as possible, double-difference measurements are the easiest and most reliable way to remove the FCBs in GPS satellites and receivers [3] [4].

On the other hand, for a user who employs a single receiver, precise point positioning (PPP) cannot be achieved by following the above methodology. To resolve the integer ambiguities when using a single receiver, single-difference measurements are used to estimate GPS satellite-dependent FCBs in advance [5]. The POD software in JAXA has two methodologies for fixing integer ambiguities: one based on the double-difference IAR and the other based on the single-difference technique.

Figure 1 shows the processing flow of the POD software. The software can estimate the orbit and clocks as well as the other parameters as shown in Table 1. These parameters can be selected whether to be estimated or to be fixed at the value entered from external files. GNSS orbits and clock biases can also be fixed using these external files (e.g. IGS final/Rapid orbits [6] or orbits previously estimated using the software).

At the first step of the POD for a LEO satellite, kinematic positions and clock biases with respect to the GPS time at each measurement epoch are estimated using GPS code measurements of the LEO receiver. The low-quality measurements are detected and removed in this step. In the next step, the kinematic positions of the LEO satellite are approximated by numerically integrating the equation of motion based on a selected force model to estimate a priori orbit of the LEO satellite as well as the dynamical orbit parameters.

After estimating the a priori orbit, clock biases, and dynamical orbit parameters of the LEO satellite, the step of orbit improvement is executed using the weighted least square method. In the orbit improvement step, integer ambiguities are estimated, which work as a constraint to the GPS carrier phase observations. Finally, an improved orbit may be obtained and validated using residuals of GPS measurements as well as those of SLR if SLR observation files (e.g. CRD [7]) have already been entered.

Table 1 Estimation Parameter (Selectable)

GPS / LEO satellite orbits and clock biases
Station coordinates
Earth rotation parameters
Troposphere parameters (piece-wise liner of ZTD and Gradients)
Ambiguity parameters (Integer or Float solutions)
Fractional Cycle Biases (Single-Difference of WL and NL)
Atmospheric drag adjustment parameter (Cannon-ball, Multi-surface)
Solar Radiation Pressure parameters (Cannon-ball, Multi-surface, etc)
Empirical accelerations (piece-wise constant of RTN directions)
Phase Center Variations of LEO satellite
Other parameters (Geocentric offset)

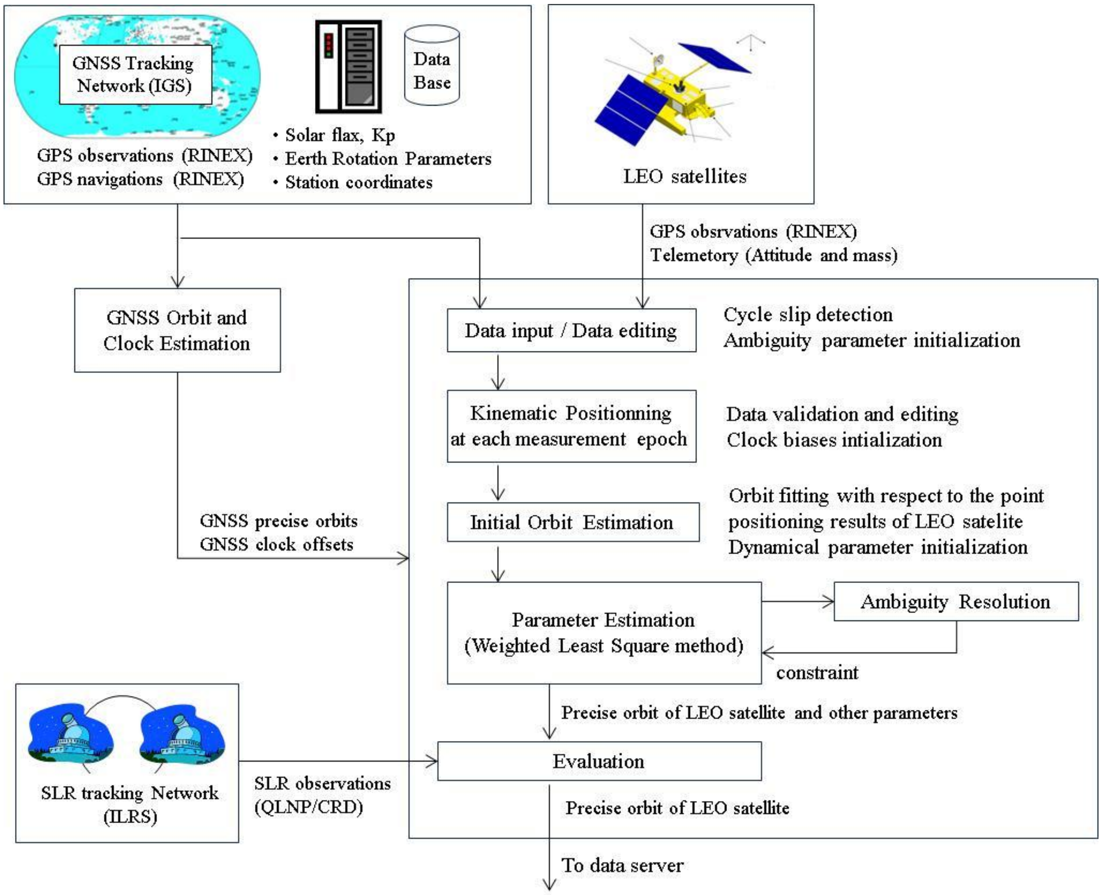

Fig. 1 POD software processing flow

2.2. GPS observation model

The observation models for the ionosphere-free combination of GPS carrier-phase and pseudo-range observations from receiver k to satellite i, in unit of length, are as follows:

$$P_{ck}^{i} = r_k^i + c\left(dt_k - dt^i\right) + T_k^i + \varepsilon_{P_c} \qquad (1)$$

$$L_{ck}^{i} = r_k^i + c\left(dt_k - dt^i\right) + T_k^i + \lambda_c w_{ck}^{i} + \lambda_c b_{ck}^{i} + \varepsilon_{\phi_c} \qquad (2)$$

where P_{ck}^{i} and L_{ck}^{i} are the ionosphere-free observations of pseudo-range and carrier-phase in frequency band c with corresponding wavelength λ_c . r_k^i is the geometric delay from GPS satellite i to receiver k. The second terms on the right sides of the equations represent a clock bias. While the clock bias of the GPS satellite can be assumed to be known with the GPS ephemeris and clock products previously estimated, the receiver clock biases are essentially unknown and have to be estimated at each epoch as part of the orbit determination process. T_k^i is the tropospheric delay, the term of $\lambda_c w_{ck}^{i}$ is phase wind-up delay and b_{ck}^{i} is the carrier phase ambiguity represented by the following equation.

$$b_{ck}^{i} = \frac{1}{f_1^2 - f_2^2}\left(f_1^2 b_{1k}^{i} - f_1 f_2 b_{2k}^{i}\right) \qquad (3)$$

where b_{1k}^{i} and b_{2k}^{i} are carrier phase ambiguities in each frequency band, which are defined by the integer number of phase cycle n_{mk}^{i} and FCB $\Delta\phi_m$ in a GPS satellite or receiver.

$$b_{mk}^{i} = n_{mk}^{i} + \left(\Delta\phi_{mk} - \Delta\phi_m^i\right) \quad \left(m = 1, 2\right) \tag{4}$$

These FCBs can be removed using the double-difference observations derived from two pairs of a GPS satellite and a receiver as follows:

$$b_{ckl}^{ij} = \frac{1}{f_1^2 - f_2^2}\left(f_1^2 n_{1kl}^{ij} - f_1 f_2 n_{2kl}^{ij}\right) \tag{5}$$

Thus, in order to estimate the carrier phase ambiguity of ionosphere-free combination precisely, the number of phase cycles in each frequency should be estimated as an integer value. A method to resolve the double-difference integer ambiguity is commonly used to decompose the ambiguities into wide-lane and narrow-lane ambiguities. For more detail on the method, refer to the [3] [4].

2.3. Dynamical model

The motion of a satellite can be obtained from given initial conditions and given models of the acceleration as a function of time t, satellite position r, and velocity v using a step-wise numerical integration. Table 2 shows examples of gravitational and non-gravitational forces for LEO satellite implemented in the POD software. The EGM 2008 with 70 × 70 subset gravity model is employed in all test cases.

A well-known reduced dynamic approach, which complements imperfect dynamics with empirical accelerations, is applied to the precise orbit determination of LEO satellites [8]. Accelerations due to the atmospheric drag and solar radiation pressure are calculated using simple cannon-ball model assuming uniform surface properties. In order to compensate for the un-modeled atmospheric drag and solar radiation pressure, piecewise constant empirical accelerations in the radial, along-track and cross-track directions are estimated. In addition, the scaling factors for atmospheric drag and solar radiation pressure are further estimated to be used to compensate for the mean value of the un-modeled accelerations in the POD software.

Table 2 Dynamical models for LEO satellite

Item	Description
Gravity Field	EGM2008 70 ×70 (selectable)
Tide	Rate, Solid earth tide, Ocean tide and pole tide corrections by IERS 2010 conventions
Third-Body Gravity	Sun, Moon, Jupiter and Venus Planetary ephemeris by JPL DE421
Atmospheric Drag	Satellite model: Cannon-ball model Atmosphere Density: NRL MSIS-E00 or JB2008
Solar Radiation Pressure	Satellite model: Cannon-ball model Shadow model: Earth and Moon
Empirical Acceleration	Acceleration model: white or first-order Gauss–Markov process Piece-wise constant for 6 min interval of radial, along-track and cross-track directions

3. LEO POD Results

In order to evaluate the POD software, orbit determination tests were conducted using actual flight data from the GRACE mission and validated their results using reference orbits provided by the Jet Propulsion Laboratory (JPL). The GRACE satellite is suitable for the POD analysis in terms of quality of the GPS observations and availability of SLR observations as well as their precise ephemeris provided from JPL [9].

The analysis period was one year from 30th September, 2011. In all cases, in this analysis, GPS precise ephemeris and clock biases were fixed to the IGS final orbit and IGS high-rate clock products, respectively [6]. Moreover, the ground GPS observations at about forty International GNSS Service (IGS) stations shown in Fig.2 were processed with integer ambiguity fixing procedure.

Table 3 GRACE-A evaluation condition

Evaluation period	1 year from 30th September, 2011
Time span (1 arc)	6+24+6 hours
GPS data interval	60 sec
No. of stations for IAR	refer to Fig.2
Coordinate frame	IGS-08
GPS orbits	IGS final orbit
GPS clock biases	IGS 30 clock
GRACE-A observation	GPS1B products
GRACE-A attitude	SCA1B products
GRACE-A ephemeris	GNV1B products

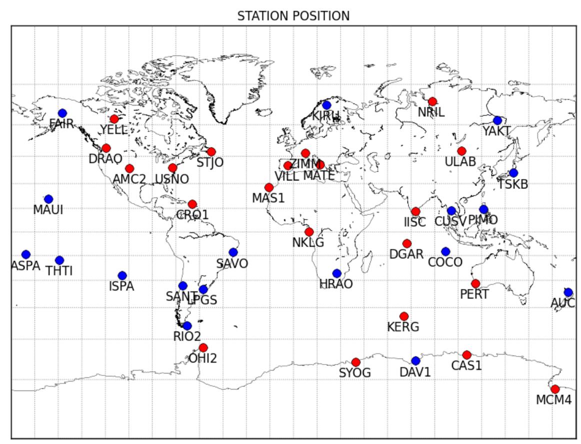

Fig. 2 IGS station map for Integer Ambiguity Resolution

3.1. GRACE-A POD Results with Ambiguity Fixing Procedure

The results from least-squares orbit determination based on ionosphere-free dual-frequency code and carrier-phase measurements of the GRACE-A satellite are shown in Fig. 3. The above figure shows errors in the POD GRACE-A orbits when compared to the precise ephemeris derived from JPL products. According to the Figure, the typical position root-mean-square of differences of these GRACE-A orbits were between 1 and 2 cm (3D rms) on a year-round basis. Further, independent SLR measurements during the evaluation period were used to compare the POD results with the calculated ranges between the GRACA-A satellite and the SLR ground stations. In this evaluation, low-quality tracking data were removed from a subset of the SLR measurements. The SLR residuals shown in the Fig. 3 (bottom) were also between 1 and 3 cm (rms). These results indicate that the POD software developed by JAXA can estimate the orbit of GRACE-A satellite with an accuracy of about a few centimeters.

In order to precisely understand the benefit of using the integer ambiguity resolution, Figure 4 shows the daily position error in comparison with the JPL solutions (above) and SLR residuals (bottom) of GRACE-A WITH and WITHOUT integer ambiguity resolution for three months. The definite improvements of the orbit accuracy of GRACE-A were found in almost all of the evaluation epochs. These results demonstrate that the POD software can estimate the integer ambiguities accurately.

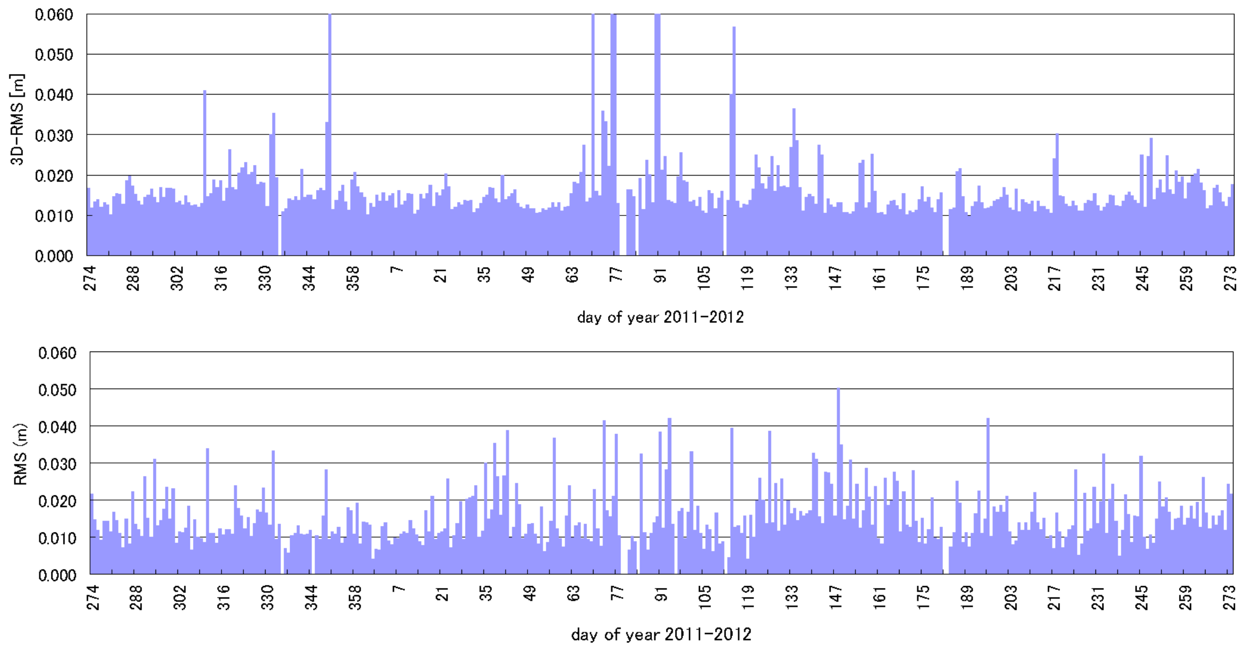

Fig. 3 Daily position error in comparison with the JPL solutions (above) and SLR residuals (bottom) of GRACE-A with IAR on a year-round basis

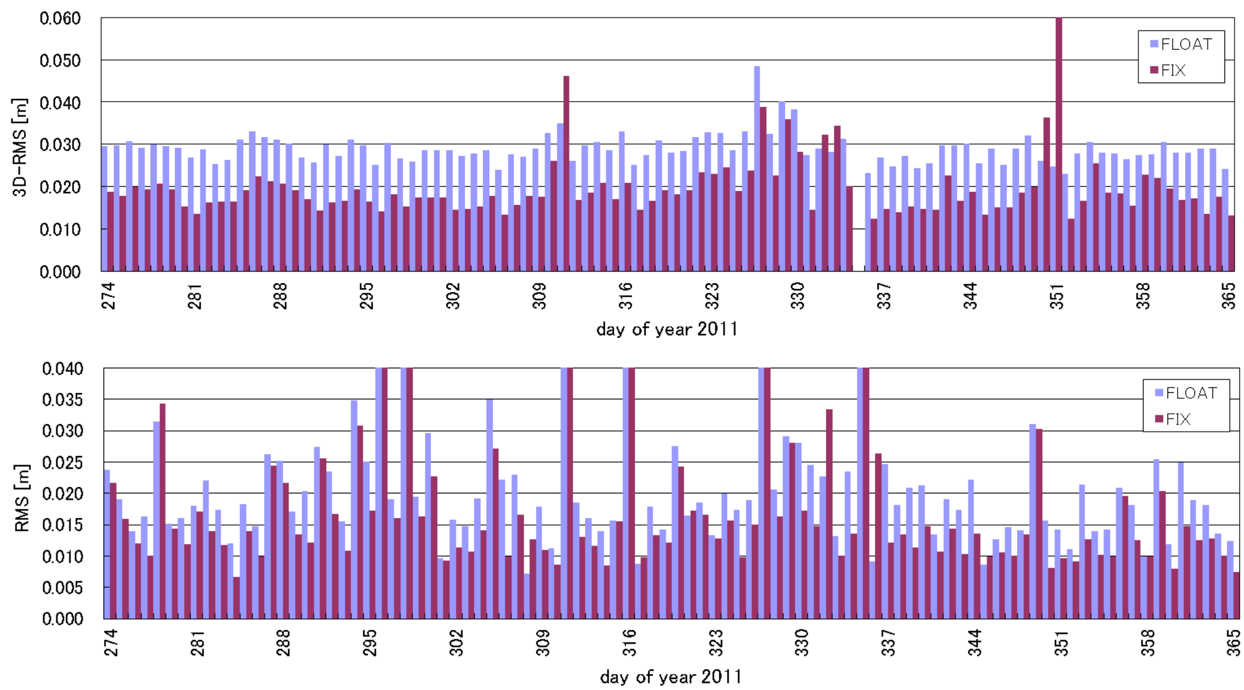

Fig. 4 Daily position error in comparison with the JPL solutions (above) and SLR residuals (bottom) of GRACE-A WITH and WITHOUT IAR for 3 months

3.2. GRACE-A POD Results with Empirical PCVs

In order to achieve the further improvement of the orbit accuracy of the GRACE-A satellite, empirical PCVs were estimated from the actual flight data on a year-round basis with the POD software. The POD strategy and estimation condition were the same as shown in the previous section.

In this study, two types of methods were demonstrated to obtain the empirical PCVs derived from reduced-dynamic solutions. The first method is a residual approach, which plots the mean value of the GPS carrier phase residuals derived from the daily POD results with respect to each azimuth-elevation grid point. Figure 5 (left) shows the empirical PCVs of the GRACE-A obtained with this residual approach. The stripe pattern has a good similarity to the pattern presented in other results of the GRACE-A antenna [10]. On the other hand, Figure 5 (right) shows the PCVs derived from a direct approach, which is the second method that estimates the PCVs as global parameters when processing the GPS carrier phase measurements. In the POD software, the distribution of the azimuth and elevation-dependent PCVs are presented as spherical surface harmonics.

$$\varphi(\alpha, z) = \sum_{n=1}^{n\max} \sum_{m=0}^{n} \tilde{P}_{nm}(\cos z)\left[a_{nm}\cos(m\alpha) + b_{nm}\sin(m\alpha)\right] \quad (6)$$

where $\varphi(\alpha, z)$ is the azimuth and elevation-dependent PCV, $\tilde{P}_{nm}$ is the Legendre polynomials of degree n and order m, and a_{nm} and b_{nm} are the coefficients which describe the dependence on the PCVs' distributions. In this study, the degree and order of the coefficients were set to twenty, which made the stripe pattern fainter than with the residual approach.

Figure 6 shows the rate of improvement of the GRACE-A orbit in comparison with the JPL solutions corrected by the empirical PCVs obtained from the direct approach. According to the result, the accuracy of the GRACE-A orbit was improved by about 5 to 10 percentage. This result indicates that the application of empirical PCVs estimated by the POD software has beneficial effects on the orbit accuracy of LEO satellites.

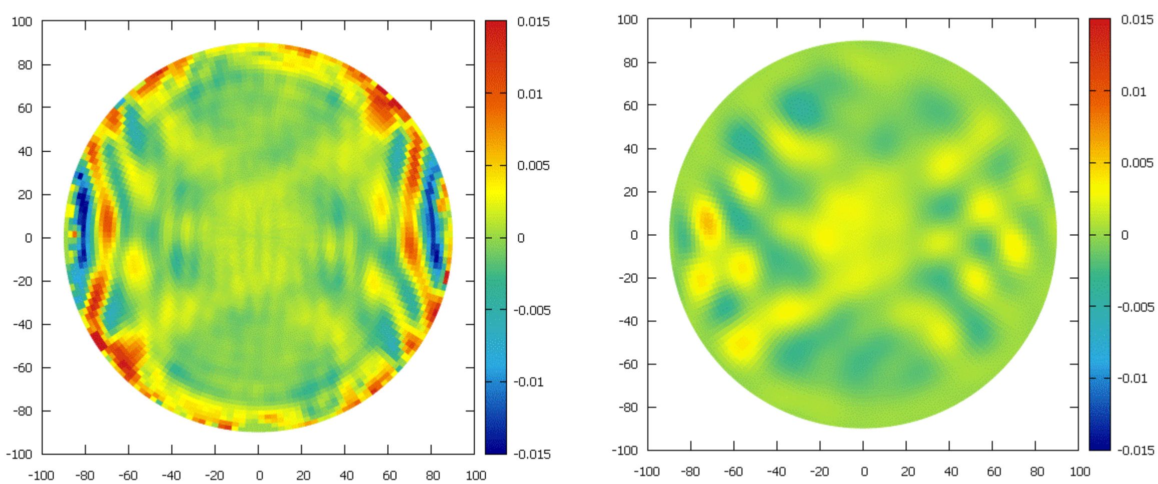

Fig. 5 Empirical PCVs of GRACE-A antenna in meters with residual approach (left) and direct approach (right) on a year-round basis from 2011 to 2012

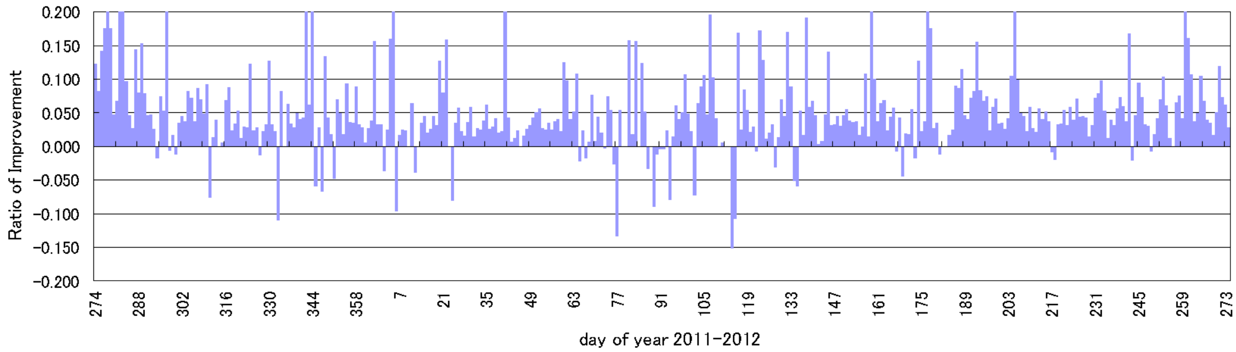

Fig. 6 Rate of improvement of the GRACE-A orbit in comparison with the JPL solutions and SLR residuals (bottom) corrected by the empirical PCVs (direct approach)

4. Conclusion

JAXA has developed the GPS-based POD software for LEO satellites implementing the ambiguity fixing procedure and empirical PCVs corrections. The evaluations of the POD software were conducted using actual flight data from the GRACE mission and the evaluation results were validated using reference orbits provided by the JPL and SLR observations. The typical position root-mean-square of differences of GRACE-A orbit were between 1 and 2 cm (3D rms), and the SLR residuals were also between 1 and 3 cm (rms) on a year-round basis with integer ambiguity fixing procedure. These results indicate that the POD software developed by JAXA can estimate the orbit of GRACE-A satellite with an accuracy of about a few centimeters. For further improvement of the orbit accuracy of the GRACE-A satellite, empirical PCVs were estimated from the actual flight data on a year-round basis with the POD software. The GRACE-A POD results indicate that the application of empirical PCVs estimated by the POD software has beneficial effects on the orbit accuracy of LEO satellites.

5. References

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