RADIATION OPTIMUM SOLAR-ELECTRIC-PROPULSION TRANSFER FROM GTO TO GEO

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Abstract: *If a spacecraft is transferred from GTO to GEO with electric propulsion the thrust is usually so low that the transfer takes a few months. During that time the radiation belts are continuously crossed. This paper investigates strategies that minimize the accumulated radiation doses. By raising the apogee to above 70000 km instead of 55000 km the radiation dose can be reduced by nearly 10 %. The price to be paid is an extra 10 % in transfer time and propellant.*

Keywords: *Solar-Electric Propulsion, Radiation, Trajectory optimisation, GTO, GEO*

1. Introduction

While electric propulsion is already used for station keeping purposes, the current technological improvements make the implementation of full electric propulsion (“full-EP”) telecom satellites realistic in the near future. ESA has performed a CDF study where solar electric propulsion is applied to reach GEO. The results of the Rendezvous and Refuelling Demonstrator study can be found in [1]. As a matter of fact also in the US a lot of analysis is currently performed in this area. In [2] transfers are investigated with a minimum radiation dose accumulated during the transfer. Radiation is a major concern for such transfers that typically last 6 months.

The total fluence on the spacecraft which is accumulated during the transfer cannot be reduced by a large extent because the minimum time solution already aims at a fast orbit raise which automatically reduces the radiation doses. In literature no radiation minimisation for a low-thrust transfer from a geostationary transfer orbit (GTO) to GEO was found. However, for a LEO to GEO transfer with an inclination change of 15° such an analysis was performed in [2] and a reduction of the radiation of 3.9 % was calculated. For an inclination change of 0° no reduction of the radiation can be achieved at all. In this paper it will be analysed how much the radiation dose in a GTO to GEO transfer can be reduced.

2. Finding the optimum low-thrust transfer between two orbits

The low-thrust transfer from GTO to GEO is well understood and presented in [3] and [4]. The optimum solution is repeated here for an initial GTO of:

$H_p(t_0)$ = 250 km

$H_p(t_0)$ = 35786 km (geostationary altitude)

$i(t_0)$ = 6°

and the final GEO orbital elements:

$H_p(t_f) = 35786$ km

$H_p(t_f) = 35786$ km

$i(t_f) = 0°$

Right ascension of ascending node is 0° and argument of perigee is 180°, but both parameters will not change (no perturbations are considered). A spacecraft mass of 5 tons is assumed and a thrust level of 900 mN.

A software (described in full detail in [4]) is used to determine the optimum thrust law to transfer the spacecraft from the initial to the final orbit in a minimum amount of time.

The thrust vector is projected in a local reference frame centred in the spacecraft. The thrust is decomposed into a tangential component u_v aligned with the velocity vector, a normal component u_n, normal to the trajectory and a bi-normal component u_h, normal to the orbital plane. In this reference frame the elevation angle ϕ is defined as the angle between the thrust direction and the plane tangential to the trajectory containing u_v and u_h. In the same reference, the azimuth angle α is defined as the angle between the projection of the thrust vector in the tangent plane and the velocity vector **v** (see Fig. 1).

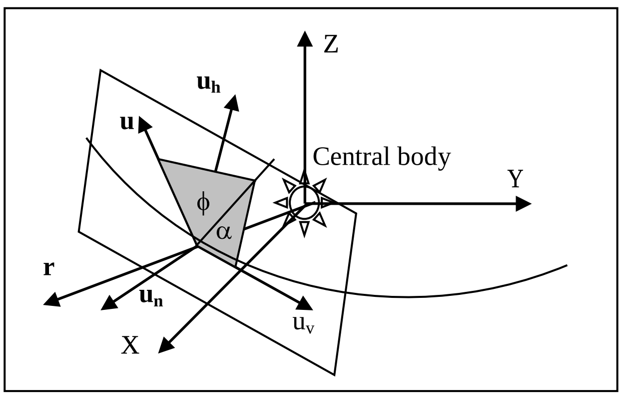

Figure 1: Definition of thrust angles elevation ϕ and azimuth α in a local reference frame.

Once the thrust direction is given, the spacecraft can roll around the longitudinal axis (direction of the thrusters) until the Sun direction is perpendicular to the axis of the solar arrays. By rotating the solar arrays a Sun-incidence angle of 90° can always be achieved.

To find the law of optimum control the maximum principle of Pontryagin is applied. This will lead to a boundary value problem which is difficult to solve in the case of more than one hundred revolutions in eccentric orbits. Therefore, two ideas are exploited, both taken from the work of Geffroy [3]:

- applying averaging methods: the right-hand sides of the differential equations are averaged over one orbital revolution. The advantage is the "smoothing" of both the state and adjoint variables, however, maintaining the oscillatory behaviour of the thrust vector. As a consequence the numerical instability of the boundary value problem disappears.

- solving a series of related (but more simple) problems: it is essential to have good initial estimates for the solution of a boundary value problem. Therefore, related problems are solved first to get better initial estimates of the adjoint variables. In Geffroy's work the following problems are solved in this sequence:

    1. Problem "P2": minimum propellant consumption without any constraint on the control vector which is the acceleration rather than the thrust vector. At first the two-dimensional (planar) problem is solved analytically and then the three dimensional problem is solved with fixed transfer time and a free final angular variable L_1.

    2. Problem "P1": minimum transfer time for the three-dimensional transfer with a limited thrust level (transfer time free, "number of revolutions" L_1 free).

    3. Problem "P3": minimum propellant consumption for a fixed transfer time.

With the solution of problem "P1", the structure of the final solution of the problem "P3" is known. Yet, the initial values of the adjoint variables differ considerably. In [3] the fact that the Hamiltonian averaged over one revolution is zero is not exploited. We use this condition to calculate the initial values of the new adjoint variables which are added in problem P1 (p_m, the adjoint variable for the mass) and in P3 (p_τ, the adjoint variable for the time variable τ). Thus, better initial estimates are obtained and one step in solving the problem P2 can even be skipped.

Furthermore, an improvement with respect to the algorithm described by Geffroy was introduced for the averaging of the differential equations which is performed by Gauss-Legendre quadrature. Instead of calling the NAG routine D01BAF at every time step and for each state variable during the solution of the boundary value problem, the weights and abscissae are calculated only once (with the NAG routine D01BBF) and then used to perform the averaging by simply adding the function values according to their weights. This change in the software reduced the CPU-time by a factor of three.

Three operations have to be performed in order to establish the boundary value problem of any of the problems above:

- Operation 1: **Minimisation** of the Hamiltonian with respect to the control variable *u* to determine the optimum control law.

- Operation 2: **Differentiation** of the Hamiltonian to obtain the differential equations governing the state and adjoint variables.

- Operation 3: **Averaging** of the differential equations (or of the Hamiltonian)

The order of the three operations needs not to be maintained as shown by Geffroy as long as the minimisation is done before the averaging. Therefore, in cases where the Hamiltonian can be averaged analytically, operation 2 and 3 are exchanged.

3. Radiation Model

A critical issue with a solar electric propulsion transfer from GTO to GEO is the radiation which the spacecraft is exposed to when it is crossing the van Allen belts. During the E-Vega CDF study [5] where it was analysed how to bring a spacecraft with electrical propulsion to GEO, a chemical manoeuvre was proposed to avoid an overlong stay in the radiation belts. Figure 2 shows the jump in the altitude at MJD=2722 when the manoeuvre is performed. Spiralling all the way through the radiation belts was considered as causing too much degradation of the spacecraft.

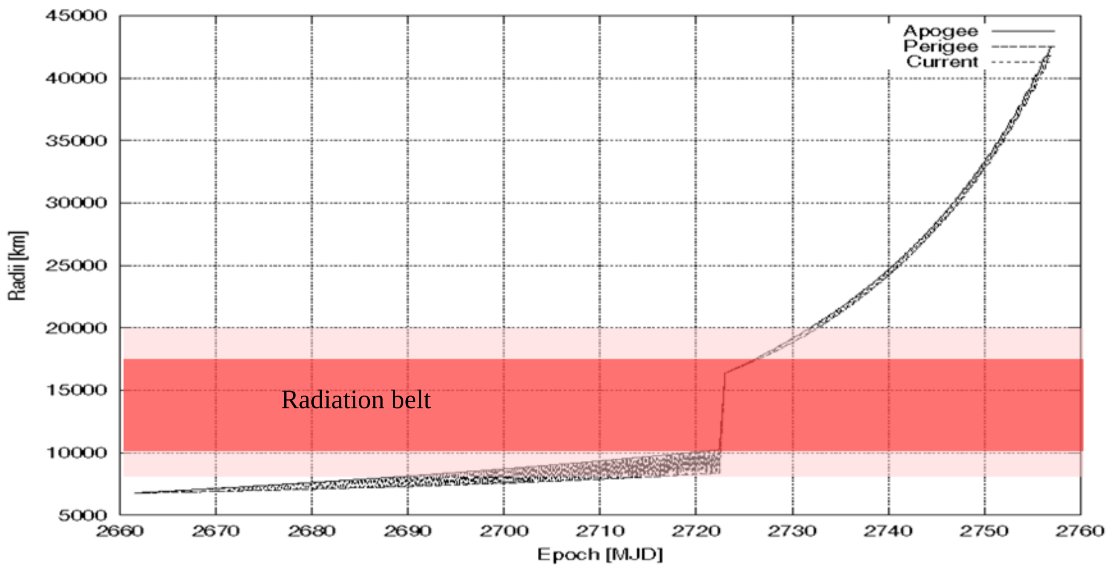

Figure 2: Proposed E-Vega altitude profile for a solar-electric transfer from LEO to GEO including one chemical manoeuvre to skip the radiation belts [5].

For the analysis in this paper the software package SPENVIS [6] is used to produce a model for the radiation doses. SPENVIS is a www-based tool intended to facilitate the use of models of the spatial environment in a consistent and structured way. The SPENVIS system consists of an integrated set of models of the space environment, and a set of help pages on both the models and the SPENVIS system itself. The NASA AP-8 and AE-8 models [7] were chosen with the option “solar maximum”. They consist of maps that contain omnidirectional, integral electron (AE maps) and proton (AP maps) fluxes in the energy range 0.04 MeV to 7 MeV for electrons and 0.1 MeV to 400 MeV for protons in the Earth's radiation belt.

Figure 3 shows the proton flux as function of orbital altitude for three energy levels. A 172-day transfer of a spacecraft of 5 tons and a thrust level of 900 mN was simulated. It can be seen that the peak radiation of 0.1 MeV protons is at an altitude of about 15 000 km, of 1 MeV protons at about 12 000 km and of 10 MeV protons at about 5 000 km.

Figure 4 shows the proton fluence (flux integrated over time) for the three different energy levels. Since the peak of the 10 MeV radiation is at 5000 km altitude, the total radiation dose at this energy level is basically reached after 60 days. Thereafter the additional radiation dose is comparably small. At an energy level of 0.1 MeV 99 % of the total radiation is accumulated during the first 120 days.

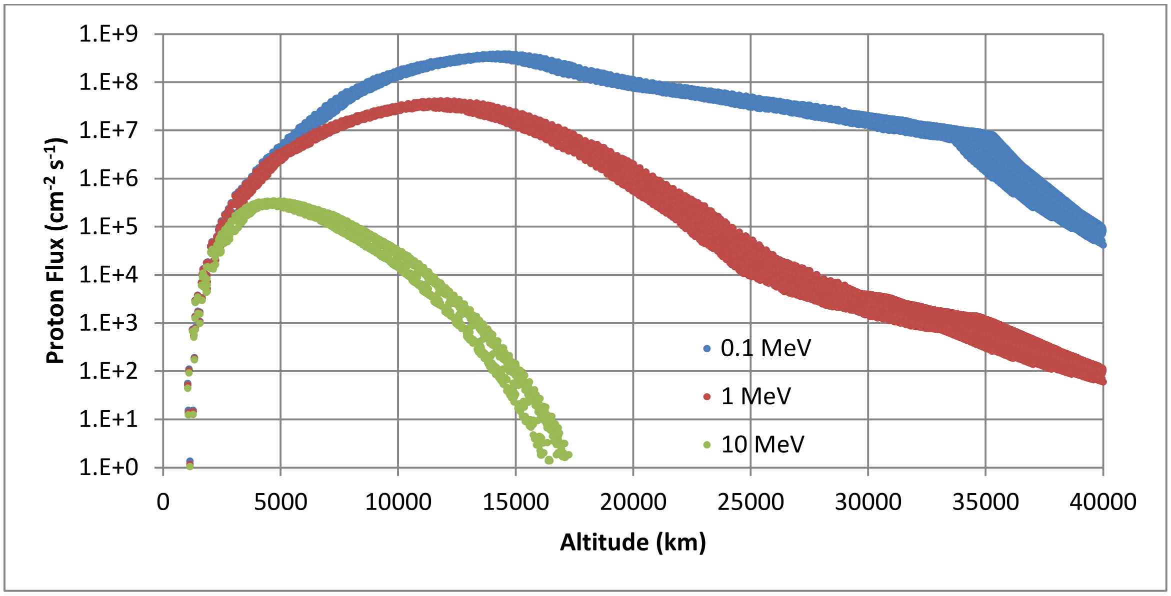

Figure 3: Proton flux as function of altitude during a 172-day SEP transfer from GTO to GEO

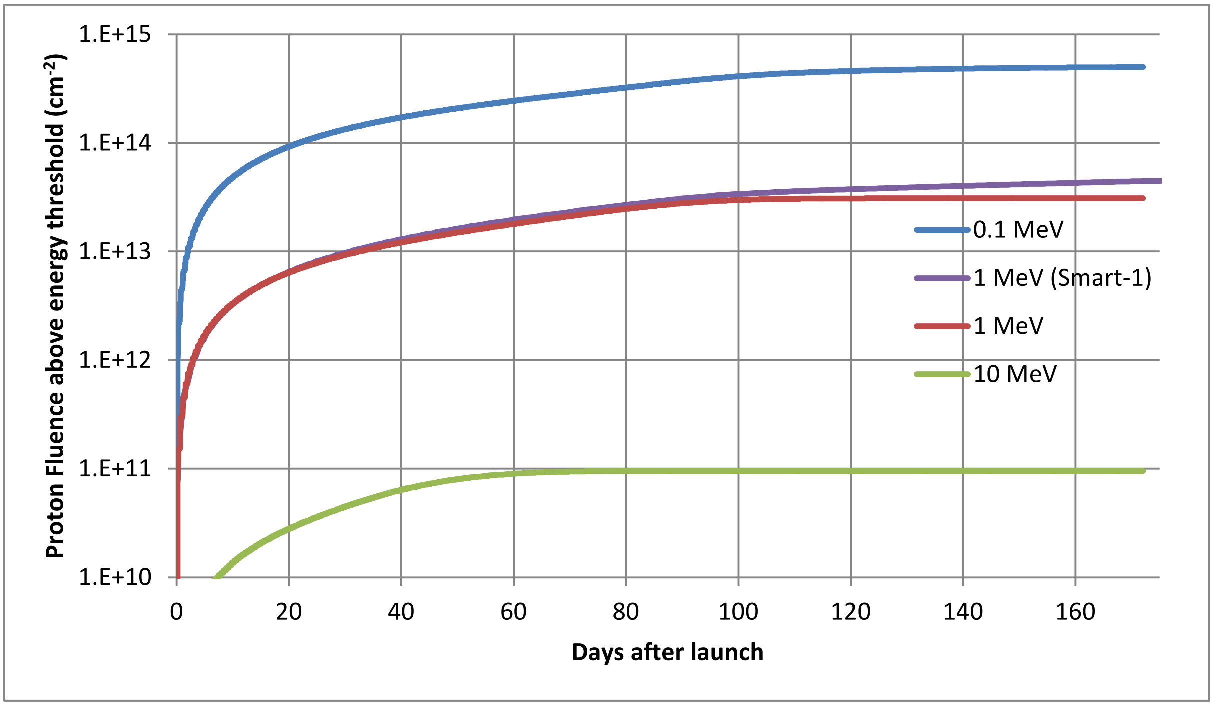

Figure 4: Proton fluence during a SEP transfer from GTO to GEO (including simulated fluences for the Smart-1 trajectory)

Since the minimisation of the radiation doses will require the calculation of thousands of different transfers, a very simple radiation model is needed in the software. Therefore, the radiation dose is approximated by integration over time of a simple function depending only on altitude *h*:

$$f_{rad} = 3.3 \cdot 10^{7} \exp\left(- (h - 11700)^{2} / 2 \cdot 10^{7} \right)$$

The function f_{rad} was determined by a fit to the 1 MeV flux calculated with SPENVIS for the GTO to GEO transfer. Figure 5 shows that the function f_{rad} provides a good fit for the altitude range where the peak flux is encountered.

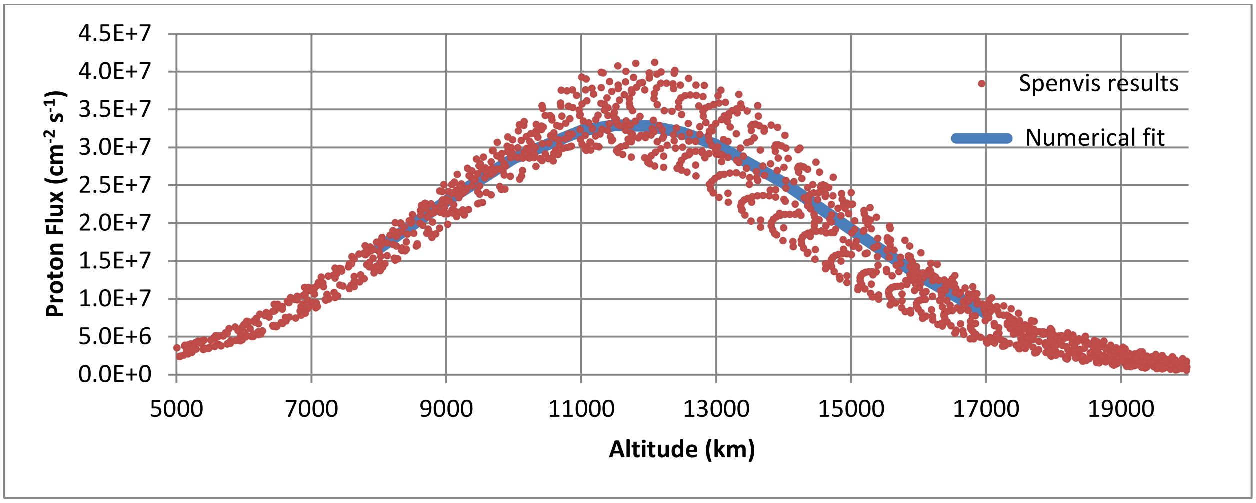

Figure 5: Fit of the 1 MeV proton flux predicted by SPENVIS [6] for a GTO to GEO transfer.

4. Reducing the radiation dose in the GTO to GEO transfer

A strict mathematical minimisation of the radiation fluence is very complicated. However, with the state-of-the-art desktop computers, very complex optimisation problems can nowadays be solved in a reasonable amount of time. The software developed for Smart-1 to optimise the multi-revolution low-thrust transfer from one orbit to another [4] was re-programmed as a subroutine which can be called in an optimisation package like SNOPT [8].

The idea is to split the transfer trajectory in two parts. $X = (hp^i, ha^i, i^i)$ is the intermediate orbit where the two parts of the trajectory merge. X is determined by the perigee radius hp^i, the apogee radius ha^i and the inclination i^i. Let $T_1(X)$ be the minimum time transfer solution from GTO to X and $T_2(X)$ be the minimum time transfer solution from X to GEO. These transfers are calculated in a subroutine together with an approximation of the radiation.

Having a subroutine that determines the total transfer time and the fluence of the solutions $T_1(X)$ and $T_2(X)$ it is now possible to determine X for which

$$\text{Fluence}\,(T_1(X)) + \text{Fluence}(T_2(X)) = \text{minimum}$$

SNOPT was used to solve this minimisation problem, because it has a fast convergence in spite of a low number of function calls. One difficulty in this optimisation problem was the choice of the required precision for solving $T_1(X)$ and $T_2(X)$. If the required precision is too high, this would cause excessive CPU-times, if it is too low, the numerical differentiation which SNOPT performs to find the gradients would not work, because the results of the function calls would not be reliable enough and would give wrong gradients. Some experimenting with the tolerance values was necessary but in the end the following solution was found:

$$X = (24140 \text{ km}, 73360 \text{ km}, 1.763°)$$

The total transfer time is only 10 % more (189.2 days) but the 1 MeV proton fluence is reduced by 9.5 %. Table 1 provides a comparison between the solution of the minimum time transfer problem and the solution minimising the time spent at the critical radiation altitudes.

Table 1: Comparison of key parameters of a minimum time and minimum radiation transfer from GTO to GEO.

	Minimum Time	Minimum Radiation	Difference
1 MeV Fluence (10^{14} cm^{-2})	0.3124	0.2827	-9.5 %
Transfer Time (days)	172.5	189.2	+9.7 %
Delta-V (km/s)	2.2019	2.4206	+9.9 %

The key difference in the two scenarios is the faster increase in the apogee height in case of the radiation minimisation. Having a higher apogee leads to a faster passage through the radiation belts. Another effect is the increased orbital period which reduces the total number of passages through the radiation belts. After 100 days the perigee radius is in both cases at about 22000 km (altitude=15600 km), but the spacecraft has completed only 152 orbits in the minimum radiation scenario whereas it has completed 161 revolutions in the minimum time scenario. The profile of the inclination change is very similar for both scenarios. Figure 6 illustrates the different behaviour of the orbital elements.

In order to achieve a faster increase in the apogee height, the spacecraft has to thrust close to the velocity direction along the whole orbit. Figure 7 shows the time history of the thrust direction for both scenarios. It can be seen that the elevation and azimuth remain very small during the first part of the transfer for the radiation minimum scenario. The spacecraft is basically accelerating all the time. At the end the resulting additional “over-shooting” of the apogee altitude has to be compensated which costs the extra 10 % in time and fuel.

Figure 8 is a zoom into Fig. 7 showing the first day of the transfer. Again it can be seen that elevation angles are smaller in the radiation minimisation scenario, meaning a more tangential thrust direction than in the minimum time transfer.

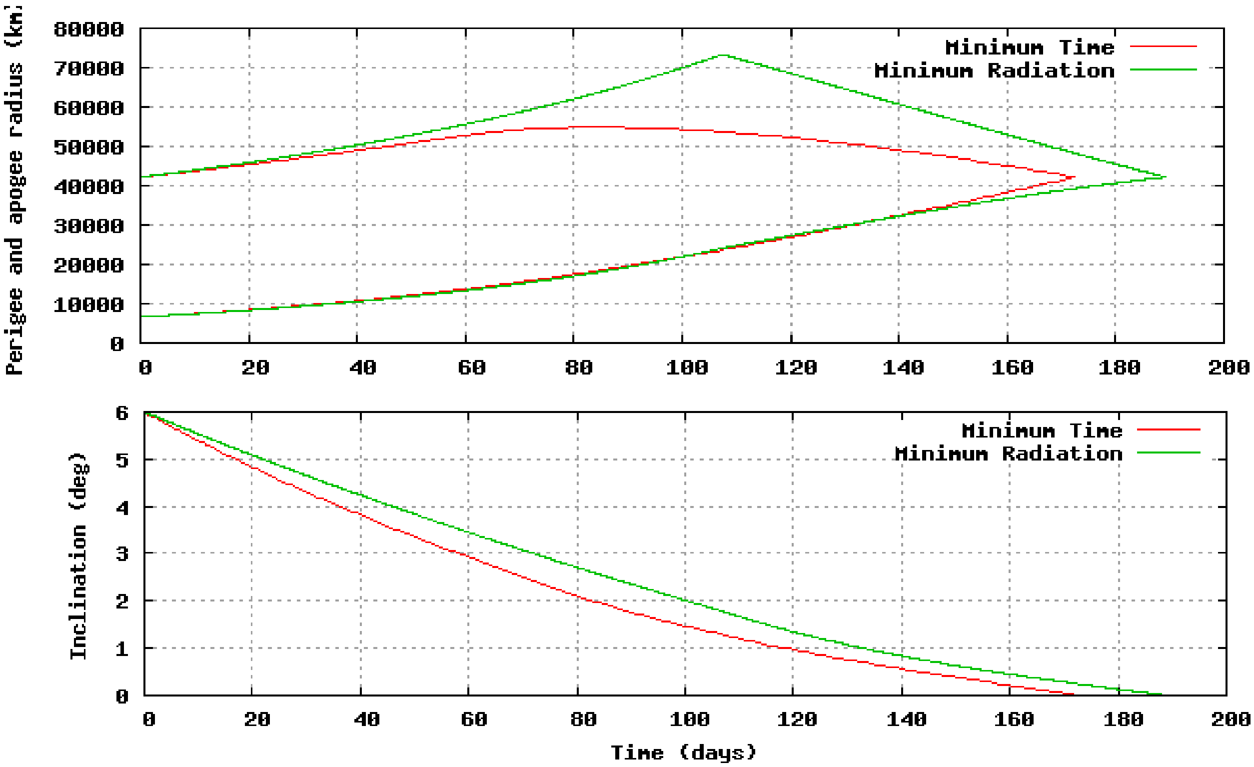

Figure 6: Inclination, perigee and apogee altitude evolution for a minimum time and for a minimum radiation transfer from GTO to GEO.

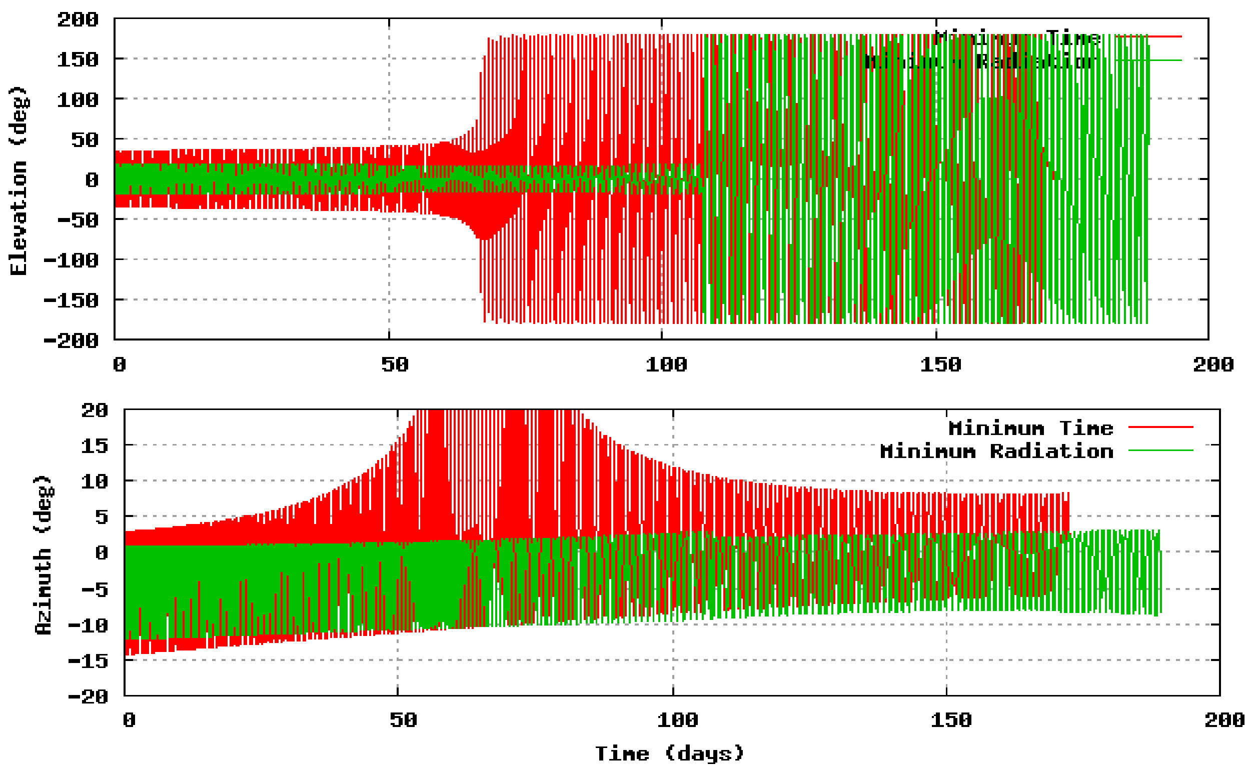

Figure 7: Time history of elevation (= in-plane component, top panel) and azimuth (=out-of-plane component, bottom panel) of the thrust vector during a minimum time transfer and for a minimum radiation transfer from GTO to GEO.

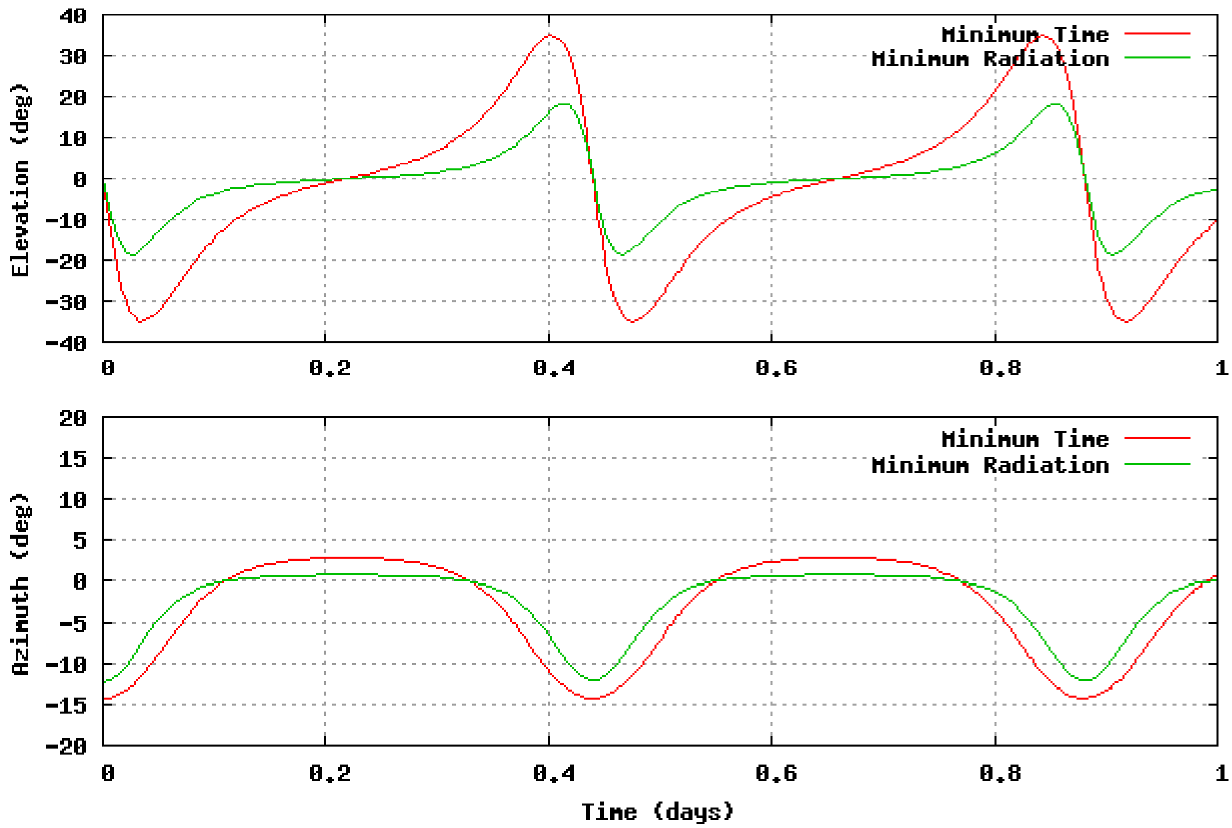

Figure 8: Time history of elevation (= in-plane component, top panel) and azimuth (=out-of-plane component, bottom panel) of the thrust vector during the first two orbits of the two different transfers from GTO to GEO (t=0 is at apocentre, t=0.22 days is at pericentre after half an orbit).

5. Conclusions

By splitting the transfer from GTO to GEO in two parts and solving the minimum time transfer problem of both parts, it was possible to use an optimisation software that found solutions with a lower radiation dose during the transfer. If the thrust is pointed closer to the tangential direction during the first part of the transfer, the apogee is raised 15000 km more than in the solution for the fastest transfer. This increases the eccentricity and thus the velocity with which the spacecraft crosses the radiation belts. It also increases the orbital period which reduces the total number of radiation belts crossings. However, this increased overshooting of the apogee has finally to be compensated with more breaking burns at the perigee to bring the apogee altitude back to 36000 km. The optimisation method presented in this paper lead to a reduction of the 1 MeV radiation dose by 9.5 % at the cost of about 10 % longer transfer time and propellant consumption.

6. References

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