NONLINEAR OBSERVABILITY FOR RELATIVE SATELLITE ORBITS WITH ANGLES-ONLY MEASUREMENTS

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ABSTRACT

Relative navigation based on angles-only measurements has been the subject of recent investigation (e.g. References 1-3). The problem is to determine the relative orbit between a chief spacecraft and a deputy spacecraft by using the line-of-sight between the two objects, assuming that the orbit of the chief is known. Reference 1 shows that the relative orbit is unobservable from angles-only (line-of-sight) measurements when linear relative orbital dynamics are assumed, unless there are thrusting maneuvers. Reference 2 investigates observability by using a relative orbit model linearized about spherical coordinates. Reference 3 introduces the concept of partial observability to determine a basis vector representing a family of relative orbits, and an initial orbit determination technique for this method. However, all of these results are based on linear relative orbital dynamics.

In this paper, the authors derive observability criteria for the nonlinear relative orbital dynamics represented by the solutions of the two-body problem. Assuming that a chief is on a circular orbit with a prescribed orbital radius, nonlinear equations of motion for the relative orbital motion of a deputy with respect to the chief are derived. A differential geometric method, based on the Lie derivatives (Ref. 4) of the line-of-sight from the chief to the deputy, is used to derive sufficient conditions for observability. It is shown that under certain geometric conditions on the relative configuration between the chief and the deputy, the nonlinear relative motion is observable from angles-only measurements.

More explicitly, sufficient conditions for observability are derived as follows:

$$\text{(i)}\;\; \mathbf{r}\times\dot{\mathbf{r}}\neq 0 \text{ or } \mathbf{r}\times\mathbf{o}_1\neq 0,$$

$$\text{(ii)}\;\; \mathrm{Proj}[\mathbf{o}_2]\times\mathrm{Proj}[\mathbf{o}_3]\neq 0,$$

where $\mathbf{r}\in\mathbb{R}^3$ is the relative position of the deputy with respect to the chief , and $\mathrm{Proj}[\cdot]$ denotes the orthogonal projection of a vector to the plane normal to $\mathbf{r}$. The vectors $\mathbf{o}_1,\mathbf{o}_2,\mathbf{o}_3\in\mathbb{R}^3$ are defined as

$$\mathbf{o}_1 = \ddot{\mathbf{r}} - \frac{\partial\ddot{\mathbf{r}}}{\partial\mathbf{r}}\mathbf{r},$$

$$\mathbf{o}_2 = \dot{\mathbf{r}} + \omega\times\mathbf{r},$$

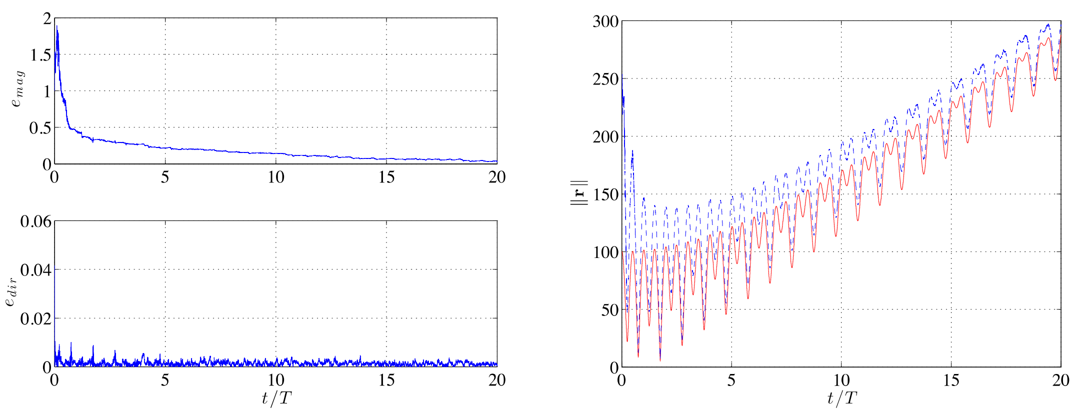
$$\mathbf{o}_3 = \mathbf{o}_1 + 2\omega\times\dot{\mathbf{r}}.$$

for the orbital angular velocity vector $\omega \in \mathbb{R}^3$ of the chief. In short, the nonlinear relative orbital motion is observable if the above two conditions (i) and (ii) are satisfied.

Several cases are identified where one of these conditions is not satisfied. These include the case when the relative position is parallel to the relative velocity observed in the local-vertical, local-horizontal (LVLH) frame; another case where the relative position is parallel to the relative velocity observed in the inertial frame; and the third case where the relative motion is co-planar with the orbital plane of the chief. But, as the given criteria (i) and (ii) are only sufficient conditions, observability is indeterminate for these three cases. Further analysis based on higher order Lie derivatives of the measurement is required to determine observability when the given criteria are violated.

To demonstrate observability of nonlinear relative orbital motion from line-of-sight measurements, we will develop several different estimation algorithms. Consider the case where the chief is on a circular orbit with an orbital altitude of 500km, and the initial condition of the deputy is chosen such that the corresponding relative motion is a 2x1 ellipse with a drift along the velocity vector of the chief. An extended Kalman filter was applied to the line-of-sight measurements. The state estimation error is represented by a normalized magnitude error and a normalized direction error as illustrated in Fig. 1a, where the direction estimation error is reduced immediately to the level of measurement noise, and the magnitude estimation error asymptotically converges. The true distance between the chief and the deputy and its estimated value are also shown in Fig. 1b.

In this paper, nonlinear observability will be explored for several cases of relative motion. This will entail both an analysis of the Lie matrix (e.g. its condition number) and investigation of the accuracy of the estimation results for the various filters designed. Conclusions will be drawn regarding the strength of observability based on such factors as the separation between chief and deputy (which drives the significance of the nonlinear effects).



(a) Estimation error in magnitudes (ratio) and direction (deg), (b) True distance (red), estimated distance (blue)

Figure 1. Estimation of Relative Orbit

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