## A Minimum-Fuel Low-Thrust Rendez-vous solved with the Switching Systems Theory

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For mission operation purposes, satellites have to make rendez-vous from their injection trajectory to their nominal operating position. Thus, it is necessary to design an accurate thrust strategy in order to fulfil the mission requirements. Let us consider a geostationary satellite equipped with electric thrusters, whose position and velocity are computed with respect to the position and velocity of a fictitious geostationary reference point, following a keplerian evolution using the Hill-Clohessy-Wiltshire framework. In this context, the minimum fuel rendez-vous problem can be recast as an optimal control problem (OCP) (see e.g. [3]).

This particular type of OCP exhibits key features that make its solution theoretically and numerically difficult to compute. These difficulties are mainly related to the nature and variety of constraints involved in the definition of the OCP: bounds on the components of the control, some window requirements ending in state constraints, non linear non convex and disjunctive operational constraints (on-off profile of the thrusters, minimum elapsed time between two consecutive firings...)

In the reference [1], an indirect method and a decomposition technique have been combined to obtain a solution satisfying all requirements for a station-keeping problem. Due to a great number of tuning parameters, satisfactory results are hard to obtain and are highly sensitive to the initialisation process.

In [2], the operational constraints are expressed as logical constraints such that the control variables are binary variables and the OCP is reduced to a linear integer programming problem.

Assuming that no singular arcs exist, it may nevertheless be shown that the minimum fuel optimal control law has an on-off profile and an ensuing difficulty is to find the optimal switching times. In the reference [4], a method for solving switched system based on the parametrisation of the switching sequence is presented. The contribution of the proposed paper is to notice that a system with an on-off control profile exhibits a switching sequence from the system whose control is off to the system whose control is on, and vice-versa. Therefore, the method from [4] may be toggled in order to refine the results obtained from [1] and [2] by optimising over the switching times.

This resolution technique is embedded in the decomposition framework proposed in [1] and considered as a third decomposition stage and is also used after the optimisation technique of [2] in order to overcome the limitations of the two previous resolution schemes. Numerical examples demonstrate the efficiency of this additional phases for the reduction of fuel consumption.

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