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Uncertainty Analysis of Mars Entry Trajectories Using Stochastic Collocation

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There are many uncertainties in the process of Mars entry, including errors on initial conditions, uncertainties in Mars atmospheric density and vehicle's aerodynamic parameters. These uncertainties pose great challenges to the control system design. To lower the difficulty in control system design and make entry trajectories more robust to the uncertainties, it is necessary to analyze the nonlinear propagation of entry trajectory uncertainty. In this paper, the propagation of uncertainties due to initial conditions, atmospheric density and aerodynamic parameters in the high-fidelity dynamic model are analyzed. The method of polynomial chaos expansions based on stochastic collocation is proposed to analyze multivariable uncertainties in entry trajectories. The evolution results of stochastic entry dynamics using stochastic collocation are compared with the results using Monte Carlo method. The comparison shows that the stochastic collocation method requires much fewer samples to achieve desired accuracy in approximating the stochastic solution. Characteristics of trajectories under uncertainties are then extensively investigated. The uncertainty analysis of Mars entry trajectories using stochastic collocation not only further expands the application of stochastic collocation method in high-dimensional nonlinear dynamics, but also provides a reference for the robust design of Mars mission.

Key Words: Uncertainty Analysis; Mars Entry; Polynomial Chaos; Stochastic Collocation

Nomenclature

r	:	radial distance from the Mars center
θ	:	longitude
φ	:	latitude
V	:	velocity
γ	:	flight path angle
ψ	:	heading angle
σ	:	bank angle
ω	:	Mars angular rate
g	:	Mars gravitational acceleration
m	:	vehicle mass
S_{ref}	:	vehicle reference surface area
C_L	:	lift coefficient
C_D	:	drag coefficient
ρ	:	Mars atmospheric density
n	:	g-load
q	:	dynamic pressure
Q	:	stagnation point convective heat load
R_n	:	nose radius
Subscripts		
0	:	initial
f	:	final
min	:	minimum
max	:	maximum

1. Introduction

Mars exploration, especially the surface exploration, has been a hot topic in recent years and attracted much attention of global scientists and engineers. The process of entry, descent and landing (EDL) is critical to the Mars landing mission due to its challenges caused by uncertain entry environment[1)]. In addition, the inaccuracy of aerodynamic model and the deviation of initial entry states make the entry process more challenging. Therefore, to make entry trajectory more robust to these uncertainties, it is necessary to analyze the effect and propagation of uncertainties in initial entry states, atmospheric density and aerodynamic parameters. The main objective of this paper is to address the problem of uncertainty analysis for Mars entry mission.

The problem of analyzing the evolution of uncertainty has been of great interest in the scientific and engineering community. In summary, there are mainly three kinds of methods for uncertainty analysis, including Monte Carlo (MC) method, linear methods and nonlinear methods[2)]. The MC usually needs a lot of samples to acquire the accurate statistic information and often acts as a baseline method to verify linear or nonlinear approximation uncertainty propagators[3-5)]. Due to the assumption of small disturbance, the linear uncertainty propagation methods work well for linear systems[6-8)], but are inapplicable for high-fidelity entry dynamic system, which is a highly nonlinear system. The method based on polynomial chaos expansion (PCE), as a nonlinear uncertainty propagation method, has gained much attention because of its good performance and easy application. The PCE method, which is first proposed by Wiener, uses polynomial series to approximate the inputs and outputs of the dynamic system[9)]. Nobile et al developed a sparse grid stochastic collocation method based on Smolyak sparse grid to reduce the required sample size in high-dimensional PCE[10)]. Fisher analyzed the stability of the stochastic dynamics using intrusive PCE[11)]. Prabhakar et al proposed a novel computational framework based on polynomial chaos to analyze the evolution of the uncertainty in state variables of the low-dimensional hypersonic flight dynamics[12]. Jones et al presented the use of PCE for the nonlinear, non-Gaussian propagation of orbit state uncertainty[13].

In this paper, the method of PCE based on stochastic collocation (SC) is proposed to analyze multivariable uncertainties in Mars entry trajectories. The multivariable polynomial chaos expansions in uncertainty parameters are formulated by using tensor products of univariate orthogonal polynomial bases, which are tailored in the Wiener-Askey scheme to achieve excellent convergence. Thus the stochastic solution to the ordinary differential equations describing the high-fidelity entry dynamics can be approximated by multivariable polynomial chaos. To solve the coefficients of the polynomial chaos, the SC method which combines the Galerkin projection and pseudospectral collocation is employed. According to the Gauss quadrature rules, nodes of every uncertainty parameter in initial condition, atmospheric density and aerodynamic coefficients are distributed. Then the tensor product grid which consists of the pregenerated nodes is generated and used for the generation of samples which require the evaluation of entry dynamic equations. The SC method allows for the usage of existing entry trajectory propagator and does not need to reformulate the stochastic entry dynamic equations every time the uncertainty parameter changes from one to another. A qualitative and quantitative analysis of the effects of parameter uncertainty on entry trajectories is also presented in this paper, which can provide a good reference for the Mars mission design.

This paper is organized as follows. In Section 2, the entry problem is formulated for the sake of completeness. In Section 3, the PCE is briefly introduced and the SC method is described. The statistics of the approximate solution obtained by the SC method is presented at the end of Section 3. In Section 4, the numerical demonstration of SC method is presented and the performance in accuracy and computational efficiency is verified by the MC method. In Section 5, the conclusion is given.

2. Problem Formulation

The three-degree-of-freedom entry dynamic model with high fidelity is considered in this paper[14]. The ordinary differential equations (ODEs) describing the entry motion over the rotating spherical Mars are given by

$$
\begin{aligned}
\dot{r} &= V\sin\gamma \\
\dot{\theta} &= \frac{V\cos\gamma\sin\psi}{r\cos\varphi} \\
\dot{\varphi} &= \frac{V\cos\gamma\cos\psi}{r} \\
\dot{V} &= -D - g\sin\gamma \\
&\quad +\omega^2 r\cos\varphi\left(\sin\gamma\cos\varphi - \cos\gamma\sin\varphi\cos\psi\right) \\
\dot{\gamma} &= \frac{1}{V}\left[L\cos\sigma + \cos\gamma\left(\frac{V^2}{r} - g\right) + 2\omega V\cos\varphi\sin\psi\right. \\
&\quad \left. +\omega^2 r\cos\varphi\left(\cos\gamma\cos\varphi + \sin\gamma\cos\psi\sin\varphi\right)\right] \\
\dot{\psi} &= \frac{1}{V}\left[\frac{L\sin\sigma}{\cos\gamma} - 2\omega V(\tan\gamma\cos\varphi\cos\psi - \sin\varphi)\right. \\
&\quad \left. +\frac{V^2}{r}\cos\gamma\sin\psi\tan\varphi + \frac{\omega^2 r}{\cos\gamma}\sin\varphi\cos\varphi\sin\psi\right]
\end{aligned}
\tag{1}
$$

where the radial distance from the center of Mars r, the longitude θ, the latitude φ, the velocity V, the flight path angle (FPA) γ and the heading angle ψ are the six variables describing the flying state of the entry vehicle. In this paper, the bank angle σ is considered as the only control variable. The drag and lift accelerations are given by

$$
\begin{cases}
D = \dfrac{1}{2m}\rho V^2 C_D S_{ref} \\
L = \dfrac{1}{2m}\rho V^2 C_L S_{ref}
\end{cases}
\tag{2}
$$

where C_L and C_D are the aerodynamic coefficients, m is the mass of the vehicle, S_{ref} is the reference surface area and ρ is the atmospheric density, which is obtained by the Mars Climate Database (MCD)[15].

During the entry process, the typical constraints imposed on the trajectory are control and path constraints. The control constraints are given by

$$
\begin{cases}
\left|\sigma(\tau)\right| \le \sigma_{\max} \\
\left|\dot{\sigma}(\tau)\right| \le \dot{\sigma}_{\max}, \tau \in [0,t] \\
\left|\ddot{\sigma}(\tau)\right| \le \ddot{\sigma}_{\max}
\end{cases}
\tag{3}
$$

where $\sigma_{\max}$, $\dot{\sigma}_{\max}$ and $\ddot{\sigma}_{\max}$ are the predefined bounds of bank angle, bank angle rate and bank angle acceleration, respectively.

The classical path constraints are g-load, dynamic pressure and the heating rate, which are given by

$$
\begin{cases}
n = \dfrac{\sqrt{L^2 + D^2}}{g_0} \le n_{\max} \\
q = \dfrac{1}{2}\rho V^2 \le q_{\max} \\
\dot{Q} = c\sqrt{\dfrac{\rho}{R_n}}V^3 \le \dot{Q}_{\max}
\end{cases}
\tag{4}
$$

where $n_{\max}$, $q_{\max}$ and $\dot{Q}_{\max}$ are the limitation of the g-load, dynamic pressure and heating rate, respectively. g_0 is the gravitational acceleration on the Mars surface, R_n is the nose radius of the vehicle and c is a constant value.

3. Uncertainty Analysis Using Stochastic Collocation

In this section, the SC method, which uses PCE to obtain the approximate solution of stochastic entry dynamics, is presented in detail. The statistic characteristics of trajectory states can then be directly obtained by the approximate solution.

3.1. Polynomial Chaos Expansion

Assuming that $\boldsymbol{Z}=\left[Z_1,\cdots,Z_d\right]^T$, in which the random variables are mutually independent with each other, is a random vector in Eq. (1) with the dimension of d, the Eq. (1) becomes a set of stochastic ODEs (SODEs) and can be rewritten as the following form in the state space.

$$\dot{\boldsymbol{x}}=f\left(\boldsymbol{x},t,\boldsymbol{Z},\sigma\right) \tag{5}$$

where $\boldsymbol{x}=\left[r,\theta,\varphi,V,\gamma,\psi\right]^T$ is the state vector of the entry vehicle. In addition, $\boldsymbol{x}\left(t,\boldsymbol{Z}\right)$, the solution to Eq. (5) is assumed to be a second process, which has finite variance.

In the framework of polynomial chaos as shown in Ref. 16), $\boldsymbol{x}\left(t,\boldsymbol{Z}\right)$ can be approximated by a set of finite polynomial series as follows

$$\boldsymbol{x}\left(t,\boldsymbol{Z}\right)\approx\hat{\boldsymbol{x}}\left(t,\boldsymbol{Z}\right)=\sum_{i=0}^{M}\boldsymbol{x}_i\left(t\right)\phi_i\left(\boldsymbol{Z}\right) \tag{6}$$

where $\phi_i\left(\boldsymbol{Z}\right)$ is the orthogonal polynomial, i and $\boldsymbol{x}_i\left(t\right)$ are the order and the coefficient of $\phi_i\left(\boldsymbol{Z}\right)$, respectively. Assuming that the maximal expansion order of each random variable is $i_{\max}$, then M, the total order of the solution can be obtained from the following equation.

$$\left(M+1\right)=\frac{\left(d+i_{\max}\right)!}{d!\,i_{\max}!} \tag{7}$$

The accuracy of the approximate solution can be improved by increasing M. It should be noted that $\phi_i\left(\boldsymbol{Z}\right)$ can be calculated by tensor product of univariate polynomial as given below

$$\phi_i\left(\boldsymbol{Z}\right)=\phi_{j_1}\left(Z_1\right)\phi_{j_2}\left(Z_2\right)\cdots\phi_{j_d}\left(Z_d\right) \tag{8}$$

where $j_k, k=1,\cdots,d$, is the order of the polynomial function $\phi_{j_k}\left(Z_k\right)$ satisfying

$$\sum_{k=1}^{d}j_k=i \tag{9}$$

3.2. Stochastic Collocation Method

From Eq. (6), it can be seen that if the coefficients of $\phi_i\left(\boldsymbol{Z}\right)$ can be obtained, the approximate solution to Eq. (5) can be acquired. Therefore, the key is to calculate the coefficients. The main principle of the SC method is using stochastic Galerkin projection and pseudospectral collocation to settle the coefficient determination problem. The general process of SC method can be summarized as below.

(1) Based on Eq. (6) and corresponding basis functions, the coefficients $\boldsymbol{x}_i\left(t\right)$ can be calculated by projecting the solution to Eq. (5) onto orthogonal polynomial space as given by

$$\boldsymbol{x}_i\left(t\right)=\frac{\boldsymbol{E}\left[\boldsymbol{x}\left(t,Z\right)\phi_i\left(\boldsymbol{Z}\right)\right]}{\boldsymbol{E}\left[\phi_i\left(\boldsymbol{Z}\right)\phi_i\left(\boldsymbol{Z}\right)\right]} \tag{10}$$

where $\boldsymbol{E}\left[\bullet\right]$ represents the expectation operator.

It should be noted that according to the orthogonality of polynomial base functions, the following property can be obtained

$$\begin{aligned}\boldsymbol{E}\left[\phi_{j_k}\left(Z_k\right)\phi_{l_k}\left(Z_k\right)\right]&=\int\phi_{j_k}\left(Z_k\right)\phi_{l_k}\left(Z_k\right)w\left(Z_k\right)dZ_k\\&=\xi_{j_k}\delta_{j_kl_k}\end{aligned} \tag{11}$$

where $w\left(Z_k\right)$ is the probability density function (PDF) of orthogonal polynomials, $\xi_{j_k}=\boldsymbol{E}\left({\phi_{j_k}}^2\right)$ is the normalization factor, $\delta_{j_kl_k}$ is the Kronecker delta function. Therefore, ξ_{j_k} and $\xi_i=\boldsymbol{E}\left[\phi_i\left(\boldsymbol{Z}\right)\phi_i\left(\boldsymbol{Z}\right)\right]=\xi_{j_1}\cdots\xi_{j_d}$ are constant values, which can be calculated before executing uncertainty analysis.

(2) Substituting the integral operation for the expectation operation, the Eq. (10) can be rewritten as

$$\boldsymbol{x}_i\left(t\right)=\frac{\int\boldsymbol{x}\left(t,Z\right)\phi_i\left(\boldsymbol{Z}\right)f_Z\left(\boldsymbol{Z}\right)d\boldsymbol{Z}}{\xi_i} \tag{12}$$

where $f_Z\left(\boldsymbol{Z}\right)$ is the PDF of $\boldsymbol{Z}$.

The pseudospectral collocation method is using some quadrature rule to approximate the integral in Eq. (12). To achieve better convergence, the quadrature rule should be selected with consideration of the distribution of random variables. If the selected quadrature rule does not match the distribution of the random variable, the total order of the approximate solution would be higher than that with the matched quadrature rule under the same level of approximate accuracy.

Using the quadrature rule, the Eq. (12) can be rewritten as

$$\boldsymbol{x}_i\left(t\right)=\frac{1}{\xi_i}\sum_{j=1}^{N}\boldsymbol{x}\left(t,\boldsymbol{Z}_j\right)\phi_i\left(\boldsymbol{Z}_j\right)\alpha_j \tag{13}$$

where α_j is the associated weight of $\boldsymbol{Z}_j$, and N is the number of required samples, which needs to evaluate the SODEs with determined collocation of $\boldsymbol{Z}$.

It should be noted that the collocation integral of Eq. (13) can be computed by either tensor grid or sparse grid. Considering the nonnested property of sparse grid, it is recommended that the tensor grid is employed when the number of random variables is less than 5, while the sparse grid should be employed when the opposite occurs.

3.3. Approximate Solution Statistics

Once the approximate solution to Eq. (5) is obtained, the statistic characteristics of trajectory states can be readily obtained by the polynomial coefficients of the solution. The mean of $\boldsymbol{x}(t,\boldsymbol{Z})$ can be obtained as given by

$$\begin{aligned}\bar{\boldsymbol{x}}(t,\boldsymbol{Z}) &\triangleq \boldsymbol{E}\left(\boldsymbol{x}(t,\boldsymbol{Z})\right)\\ &\approx \boldsymbol{E}\left(\hat{\boldsymbol{x}}(t,\boldsymbol{Z})\right)\\ &= \int\cdots\int\sum_{i=0}^{M}\boldsymbol{x}_i(t)\phi_{j_1}(Z_1)w(Z_1)dZ_1\cdots\phi_{j_d}(Z_d)w(Z_d)dZ_d\\ &= \boldsymbol{x}_0(t)\end{aligned} \tag{14}$$

The variance can be derived through the similar process, which is given by

$$\begin{aligned}\operatorname{var}\left(\boldsymbol{x}(t,\boldsymbol{Z})\right) &= \boldsymbol{E}\left(\left(\boldsymbol{x}(t,\boldsymbol{Z})-\bar{\boldsymbol{x}}(t,\boldsymbol{Z})\right)\left(\boldsymbol{x}(t,\boldsymbol{Z})-\bar{\boldsymbol{x}}(t,\boldsymbol{Z})\right)^T\right)\\ &\approx \sum_{i=0}^{M}\boldsymbol{x}_i(t)\boldsymbol{x}_i^T(t)\prod_{k=1}^{d}\xi_{j_k}\end{aligned} \tag{15}$$

Other statistical quantities of $\boldsymbol{x}(t,\boldsymbol{Z})$, e.g. the skewness and kurtosis, can also be readily acquired through their definitions and $\boldsymbol{x}_i(t)$.

4. Numerical Demonstration

This section presents the results of uncertainty analysis in a specific Mars entry scenario by using the SC method. Meanwhile the Monte Carlo (MC) method is employed as a reference.

Uncertainties in three kinds of entry parameters are considered, including the initial states, aerodynamic coefficients and atmospheric density. The nominal entry condition is listed in Table 1. The characteristics of the entry vehicle are similar to those of the Mars Science Laboratory, including the reference surface area of 15.9 m^2, the mass of 2804 kg, the nose radius of 6.476 m. For ease of simulation, the aerodynamic coefficients are simplified as constant values with the nominal drag coefficient of 1.415 and the nominal lift coefficient of 0.176. The limitations of the g-load, dynamic pressure and heating rate are 25, 12.85×10^3 Pa, and 2.2×10^5 W/m^2, respectively. For convenience, the bank angle is fixed as $15°$, which is satisfied with the control constraints shown in Eq. (3).

For convenience but without loss of generality, the initial altitude and lift coefficient are considered with uncertainties uniformly distributed in $[\text{-2000m}, \text{2000m}]$ and $[-15\%, 15\%]$, respectively. MCD is used to generate the nominal atmosphere with the date of June 1st, 2022. The uncertainty in atmospheric density is considered as a function of the altitude. A set of 100 dispersed atmospheric density profiles is shown in Fig. 1. A lot of simulation results show that the path constraints in Eq. (4) are satisfied with the bank angle of $15°$.

To better demonstrate, the propagation of trajectory under single and multiple uncertainties are analyzed respectively. In each case under single uncertainty, 10 nodes are selected to generate the grid and the total number of polynomial expansion terms is 5. That is to say using SC method, only 10 samples are needed to obtain the statistic characteristics of the trajectory. In the case of multiple uncertainties, the tensor grid is generated using 4 nodes in each stochastic variable, and only 64 samples are needed. The trajectory solution under multiple uncertainties is approximated by the orthogonal polynomial with the total order of 8. As a baseline, the convergence of MC is checked at every 500 samples, and is thought to be achieved when the change in expected value between successive evaluations is within the tolerance.

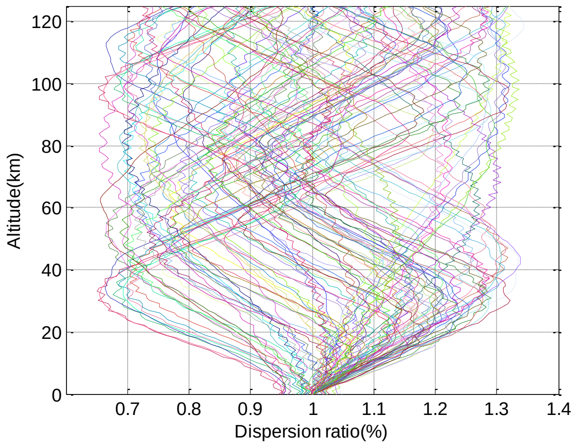


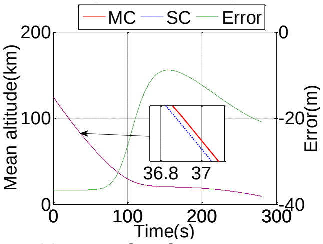
Fig. 1 Ratio of dispersed atmosphere to nominal atmosphere for 100 cases.

Table 1. Nominal entry condition.

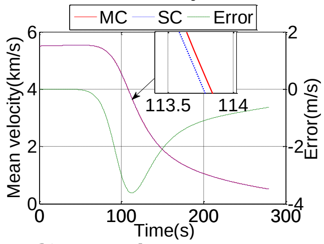
Items	Values	Units
h_0	124.56	km
θ_0	-90.072	deg
φ_0	-43.898	deg
V_0	5.505	km/s
γ_0	-13.12	deg
ψ_0	-84.19	deg

Figures 2-4 show the partial statistic results under single uncertainty in initial altitude, lift coefficient and atmospheric density, respective. The mean and variance of altitude, velocity and FPA are presented in (a)-(f), respectively. From these results, the sensitivity of the trajectory states with respect to previously mentioned parametric uncertainty can be analyzed. For example, the variance of altitude decreases first and then increases under the uniform uncertainty in initial altitude, while increases first and then decreases under the uncertainty in atmospheric density. Comparing Fig. 3 (d) with Fig. 4 (d), the effect of uncertainty in atmospheric density on the maximal value of the variance altitude is larger than that of the uncertainty in lift coefficient.

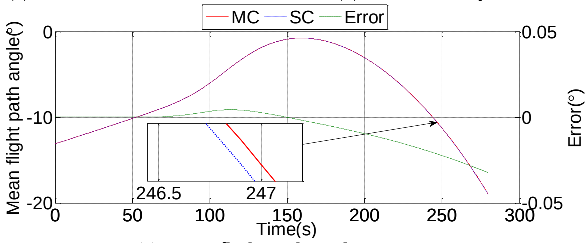
The composite effects on trajectory under multiple uncertainties are shown in Fig. 5. It can be seen from these figures that there is little error between the results obtained by MC and SC methods. The little difference shows the accuracy of the SC. Furthermore, it should be noted that in any case, the MC method needs at least 1500 samples before the convergence is reached. However, the SC method only needs 10 samples in the case of single uncertainty and 64 samples in the case of multiple uncertainties. Therefore, the SC method shows a great advantage in computational efficiency.



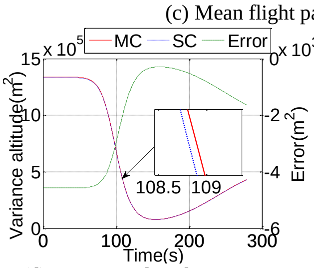
(a) Mean altitude vs. time



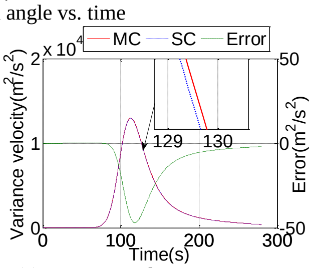
(b) Mean velocity vs. time



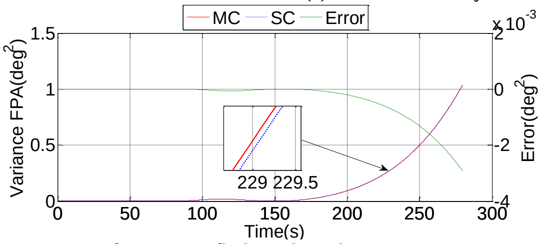
(c) Mean flight path angle vs. time



(d) Variance altitude vs. time

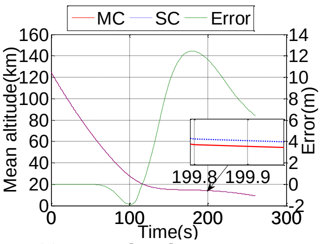


(e) Variance velocity vs. time

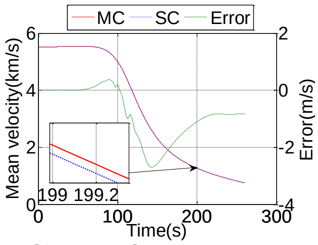


(f) Variance flight path angle vs. time

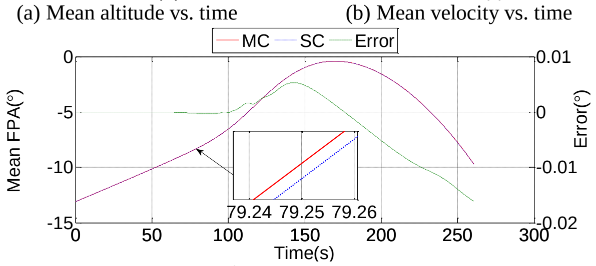
Fig. 2 Comparison of mean and variance trajectory with 2000 m uniform uncertainty in initial altitude.



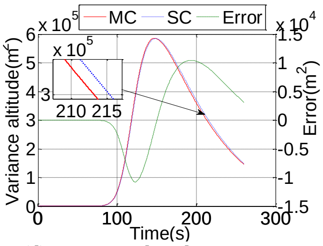
(a) Mean altitude vs. time



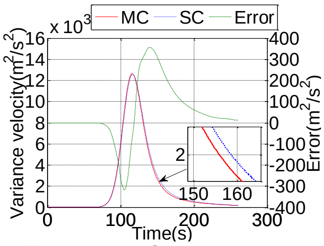
(b) Mean velocity vs. time



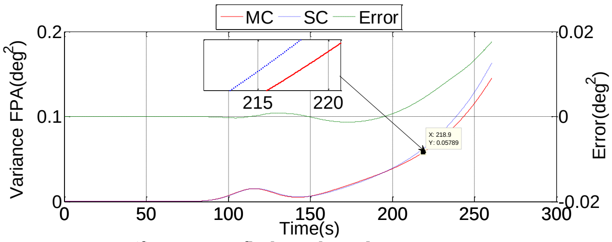
(c) Mean flight path angle vs. time



(d) Variance altitude vs. time

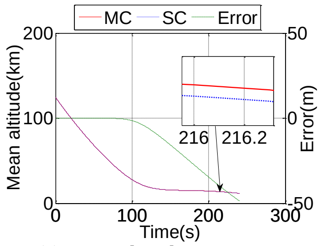


(e) Variance velocity vs. time

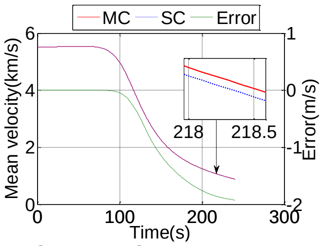


(f) Variance flight path angle vs. time

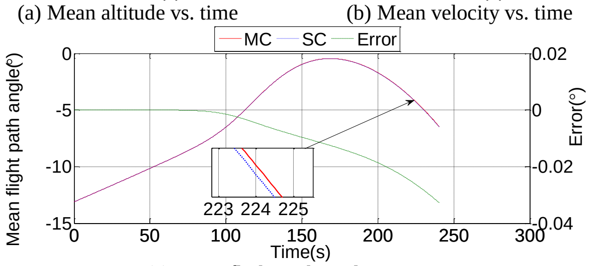
Fig. 3 Comparison of mean and variance trajectory with uncertainty in atmospheric density.



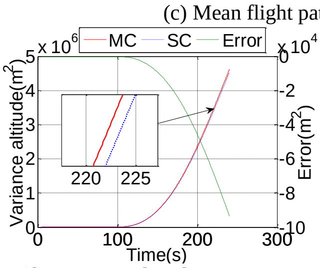
(a) Mean altitude vs. time



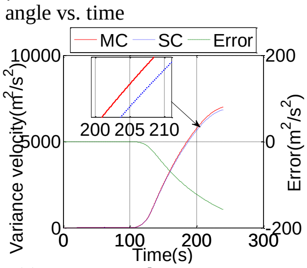
(b) Mean velocity vs. time



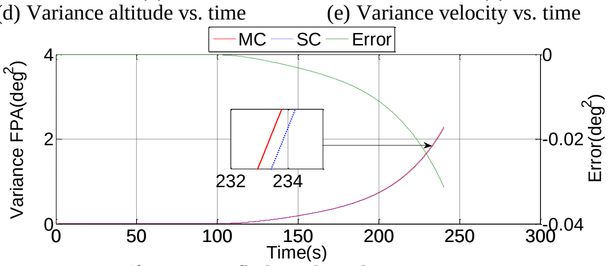
(c) Mean flight path angle vs. time



(d) Variance altitude vs. time

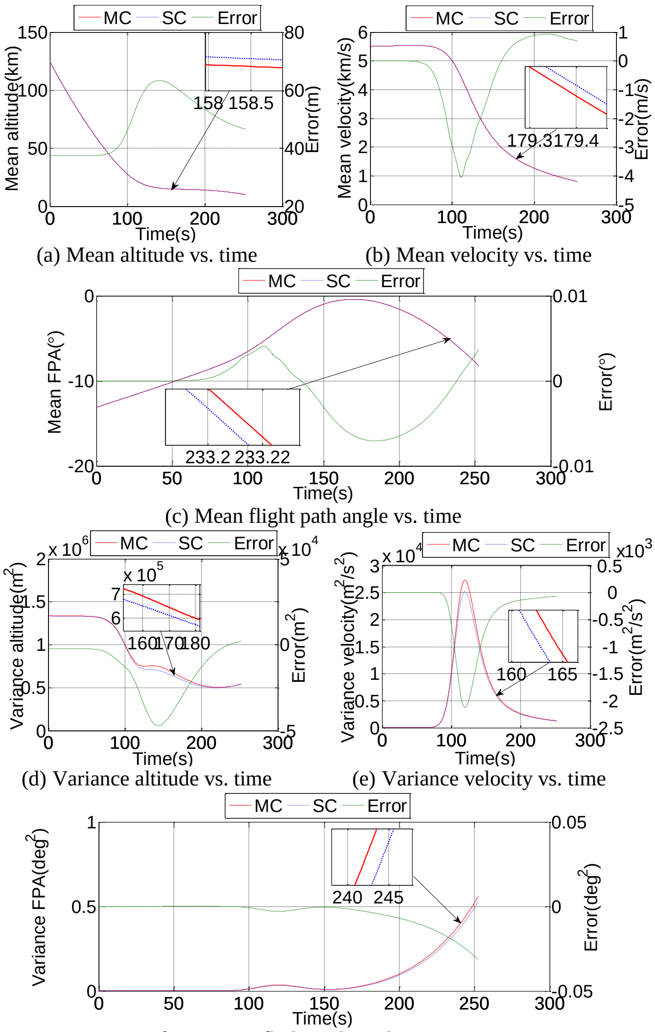


(e) Variance velocity vs. time



(f) Variance flight path angle vs. time

Fig. 4 Comparison of mean and variance trajectory with 15% uniform uncertainty in lift coefficient.



(a) Mean altitude vs. time (b) Mean velocity vs. time

(c) Mean flight path angle vs. time

(d) Variance altitude vs. time (e) Variance velocity vs. time

(f) Variance flight path angle vs. time

Fig. 5 Comparison of mean and variance trajectory with multiple uncertainties.

5. Conclusion

In this paper, an uncertainty analysis method based on stochastic collocation is presented to solve the approximation problem of the stochastic entry dynamics. The proposed method is successfully applied to the Mars entry dynamics. Monte Carlo method is employed to serve as a baseline and SC method shows great performance in accuracy and computational efficiency. The statistic characteristics of stochastic trajectory are also investigated. This paper not only develops the SC method in solving high dimensional nonlinear stochastic dynamic problem, but also provides a useful tool for Mars mission analysis.

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References

1) Braun, R. D., Manning, R. M.: Mars exploration entry, descent and landing challenges, *Journal of Spacecraft and Rockets*, **44**(2007), pp. 310-323.
2) Luo, Y., Yang, Z.: A review of uncertainty propagation in orbital mechanics, *Progress in Aerospace Sciences*, **89**(2017), pp. 23-39.
3) Ghanem, R., Yadegaran, I., Thimmisetty, C., et al.: Probabilistic approach to NASA Langley research center multidisciplinary uncertainty quantification challenge problem, *Journal of Aerospace Information Systems*, **12**(2015), pp. 170-188.
4) Chaudhuri, A., Waycaster, G., Price, N., et al.: NASA uncertainty quantification challenge: an optimization-based methodology and validation, *Journal of Aerospace Information Systems*, **12**(2015), pp. 1-25.
5) Halder, A., Bhattacharya, R.: Beyond Monte Carlo: a computational framework for uncertainty propagation in planetary entry, descent and landing, AIAA Guidance, Navigation and Control Conference, Aug. 2010.
6) Geller, D. K., Christensen, D. P.: Linear covariance analysis for powered lunar descent and landing, *Journal of Spacecraft and Rockets*, **46**(2009), pp. 1231–1248.
7) Liang L. B., Luo, Y. Z., Zhang, J., et al.: Rendezvous-phasing errors propagation using quasi-linearization method, Proceedings of AIAA Guidance, Navigation and Control Conference, Aug. 2010.
8) Saunders B. R.: *Optimal Trajectory Design under Uncertainty*, Massachusetts Institute of Technology University Press, Cambridge, 2010.
9) Xiu, D., Karniadakis, G. E.: The Wiener–Askey polynomial chaos for stochastic differential equations, *SIAM J. Sci. Comput*, **24**(2002), pp. 619–644.
10) Nobile, F., Tempone, R., Webster, C. G.: A sparse grid stochastic collocation method for partial differential equations with random input data, *SIAM J. Numer. Anal*, **46**(2008), pp. 2309–2345.
11) Fisher, J. R.: *Stability Analysis and Control of Stochastic Dynamic Systems Using Polynomial Chaos*, Texas A&M University Press, College Station, 2008.
12) Prabhakar A., Fisher J., Bhattacharya R.: Polynomial Chaos-Based Analysis of Probabilistic Uncertainty in Hypersonic Flight Dynamics, *Journal of Guidance, Control, and Dynamics*, **33**(2010), pp. 222-234.
13) Jones B. A., Doostan A., Born G. H.: Nonlinear propagation of orbit uncertainty using non-intrusive polynomial chaos, *Journal of Guidance, Control, and Dynamics*, **36**(2013) pp. 430–444.
14) D'Souza, S. N., Sarigul-Klijn, N.: Survey of planetary entry guidance algorithms, *Progress in Aerospace Sciences*, **68**(2014), pp. 64-74.
15) Millour E, Forget F, Spiga A, et al.: The Mars Climate Database (MCD version 5.1), *Lpi Contributions*, **1791**(2014), pp. 1184.
16) Xiu D.: *Numerical Methods for Stochastic Computations: A Spectral Method*, Princeton University Press, New Jersey, 2010.