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Family of Quasi-Stable Orbit around Asteroids in Strongly Perturbed Environment

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This work introduces "Quasi-Stable Terminator Orbit (QSTO)" which is a family of orbits around small bodies and has long duration. Despite the environments around small bodies are strongly perturbed by mainly solar radiation pressure, QSTO does not suffer from impact with the surface of the small body or escaping. This work shows the solution space where QSTO can exist and the usability of QSTO in numerical simulations. We investigated the solution space by also analytical method using averaging method and solving the Lagrange planetary equations in particular situations. Therefore, this work verified that analytical way to derive the region where QSTO is useful is valid by comparing numerical results and analytical results.

Key Words: Augmented Three Body Problem, Terminator Orbits, Solar Radiation Pressure

Nomenclature

r	:	distance from the origin
n	:	mean motion
a_{SRP}	:	SRP acceleration
A	:	projected area of a spacecraft
ν	:	true anomaly of the small body
a	:	semi-major axis
e	:	eccentricity
i	:	inclination
ω	:	argument of periapsis
λ	:	longitude of the ascending node
f	:	true anomaly
$\tilde{a}$	:	normalized semi-major axis
P	:	semilatus rectum of asteroid
ρ	:	density of asteroid
B	:	Unit mass of projected area

Subscripts

0	:	initial
a	:	asteroid
offset	:	offset between the origin and the focus of the orbit

1. Introduction

These days exploration of asteroid has been gathering attention all over the world because of its scientific and engineering importance. One of the asteroid exploration mission, Hayabusa2 mission, will conduct the hovering operation during staying near the asteroid.[1] Although hovering operation is conservative and safe, orbiting around the asteroid can conduct mapping the asteroid and measure gravity field of it more precisely than hovering operation. Moreover, orbiting can reduce fuel consumption compared to hovering operation. However, the dynamics around small body is so complicated due to the strong perturbation such as solar radiation pressure (SRP), tidal force and so on that it is difficult to orbit around the asteroid and designing orbit around small bodies is very restricted.

One of the most representative orbits which is stable and can keep their orbits for a long time is "Terminator Orbits" which has been investigated in some previous studies.[2-5] Although terminator orbit is stable and useful for rendezvous missions, it also has disadvantages. The terminator orbit's orbital plane must always face the sun and lack flexibility in orbit design. Moreover, since the orbital plane lies in night side, optical observation of the small body is extremely restricted.

Therefore the purpose of this work is to extend the terminator orbit concept to a family of stable orbits which includes the terminator orbit and does not suffer from impact with surface or escaping. This extended terminator orbit is called "Quasi-Stable Terminator Orbit (QSTO)" in this study. A whole solution including nonlinear region is surveyed systematically unlike some previous studies which analyzed the linearized space around a terminator orbit.[2,3] Although some previous studies proposed various types of periodic orbits around small bodies called "Resonant Terminator orbit (RTO)",[6-9] There are few analyses about quasi-periodic terminator orbits. "Quasi-Terminator Orbit" was proposed in a previous study and this orbit is quasi type of RTO. [8,9] However the analysis about the quasi type of terminator orbits is not enough. This study verified that the terminator orbit is a part of the family of stable orbits around terminator orbit. At the same time QSTO improves the flexibility of the orbit design dramatically and provides several merits the terminator orbit does not have such as optical observation of the small body.

In this work, Hayabusa 2 mission properties are used in all simulation results as an example.

2. Searching the Solution Space of Orbits around an Asteroid

2.1. Equation of Motion

Basic equation of motion is considered as an Augmented Hill 3 Body Problem (AH3BP) as shown in Eq.(1)-(3). AH3BP approximates the small body's orbit as a circular orbit and includes SRP force. These equations are defined in Hill coordinate. The origin is the center of the asteroid and X axis points in the direction of anti-solar direction. Y axis points the direction that the asteroid goes around and Z points out of plane.

$$\ddot{x} - 2n\dot{y} - 3n^2 x = -\frac{\mu}{r^3}x + a_{SRP} \tag{1}$$

$$\ddot{y} + 2n\dot{x} = -\frac{\mu}{r^3}y \tag{2}$$

$$\ddot{z} + n^2 z = -\frac{\mu}{r^3}z \tag{3}$$

SRP force a_{SRP} is expressed by Eq. (4).

$$a_{SRP} = \frac{(1+\eta)PA}{M} \tag{4}$$

Table. 1 shows parameters of Hayabusa2 spacecraft and its target asteroid Ryugu which are used in Eq. (1)-(4).

Table. 1. Properties of Ryugu and Hayabusa2

Ryugu		Hayabusa2	
Gravitational Constant μ	32 $[m^2/s^3]$	Mass M	580 [kg]
Radius R	435 [m]	Projected Area A	12.64 [m2]
Semi-major axis	1.19 [AU]	Reflectivity η	0.113

2.2. Terminator Orbits

The method to design the terminator orbits using a symmetry about equation of motions was developed in previous studies [1-4]. Figure. 1 shows variety size of terminator orbits around Ryugu. The contour shows the velocity of the orbit which the S/C has when it intersect XZ plane from –y to +y. As shown in Fig. 1, all terminator orbits face to the solar direction (-x direction) and orbital plane has some offset to the anti-solar direction (+x direction) which is expressed by x_{offset}. As the terminator orbit is far from the small body, it is close to the L2 point.

2.3 Quasi-Stable Terminator Orbits

In order to search the solution space around terminator orbit the global search was conducted. The method of global search is shown in Fig. 2. The parameters z_0 and λ_0 indicate the z value of intersection point of the terminator orbit and the initial longitude of the ascending node of it. z_0 indicates the size of orbit and λ_0 = 90 [deg] means the orbit is equivalent to a terminator orbit. The initial velocity V_0 is set to be the same value of the terminator orbit. Propagation time: 474 [day]

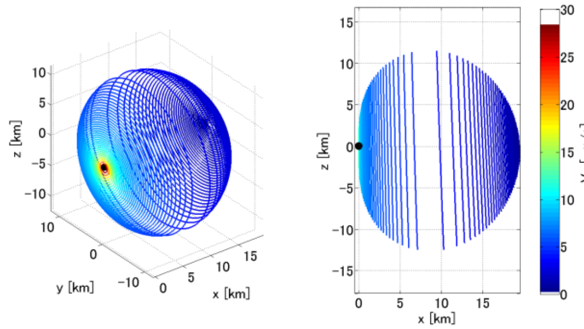


Fig. 1. Terminator Orbits.

Propagation is stopped when S/C impacts with Ryugu. The first intersection points to XY plane are plotted in Fig. 2 and Fig. 3.

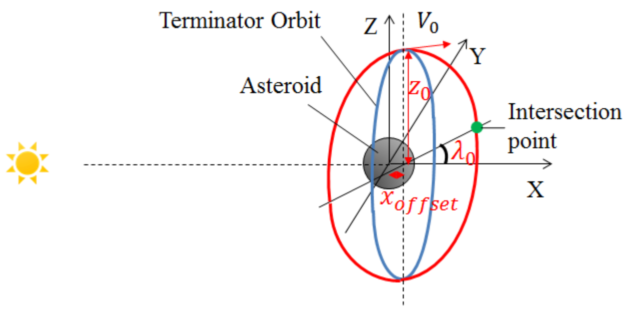


Fig. 2. Method of global search of orbits around terminator orbits.

(Ryugu's period).

Figure. 3 and 4's contour means orbits' duration and the number of orbiting respectively. Figure. 3 reveals that there are 2 areas where one keeps the S/C for long duration and the other keeps it only short duration. The number of orbiting of long duration area is also more than that of short duration area. Since all of the propagation time are 474 day, the larger the orbit becomes, the fewer the number of orbiting becomes. Figure. 5 shows some example of orbits about the plots (a)-(c) on Fig. 3. Table. 2 shows the characteristics of each type of orbit. It is found that orbit (a) keeps its orbit only approximate 10 days and finally impact with the small body. Orbit (c) is a very terminator orbit. It is found that the orbit is completely closed loop. The most interesting orbit is orbit (b) and is called "Quasi-Stable Terminator Orbit (QSTO)" in this study. This orbit looks so complicated but Fig. 5 (b)'s YZ coordinate graph reveals the S/C does not impact with the small body for a long time. Figure. 6 shows that orbit (b) for 50 days from initial state and it oscillates largely. Therefore, this type of orbit is not a periodic orbit like terminator orbit but is stable orbit which means that it can avoid from impact. This paper defines this type of orbit as QSTO. Figure. 3 also indicates that red area is equal to the existence region of QSTO.

The important thing is that extending the terminator orbit concept provides broader solution area about QSTO which can keep S/C near the small body for long duration. QSTO improves the flexibility of orbit design around the small body. Moreover, QSTO can experience sunlit side for about half time of its duration as Fig. 5 (b)'s and Fig.6's XY coordinate graph unlike terminator orbit. This characteristic is very advantageous for observing the small body using optical camera. It is verified that QSTO is useful in terms of orbit design flexibility and observing the small body.

Thus, it is should be revealed what determines this boundary which divides the solution space into impact and non-impact area. The next section analyzes this boundary by focusing on the behavior of orbital element of the orbit around asteroid.

Table. 2. Characteristics of each type of orbit

Type of orbit	(a)	(b)	(c)
Duration	10.3 [day]	474 [day]	474 [day]
The number of orbiting	3	184	185
Impact or not impact	Impact	Not impact	Not impact
Periapsis altitude	0 [m]	308 [m]	3000 [m]

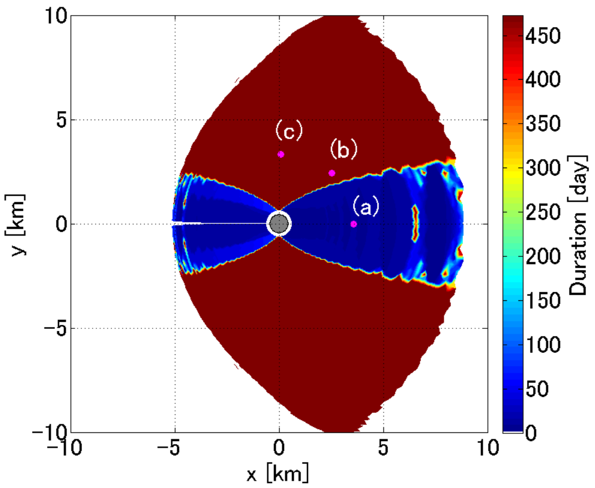


Fig. 3. Duration of Orbit around terminator orbit.

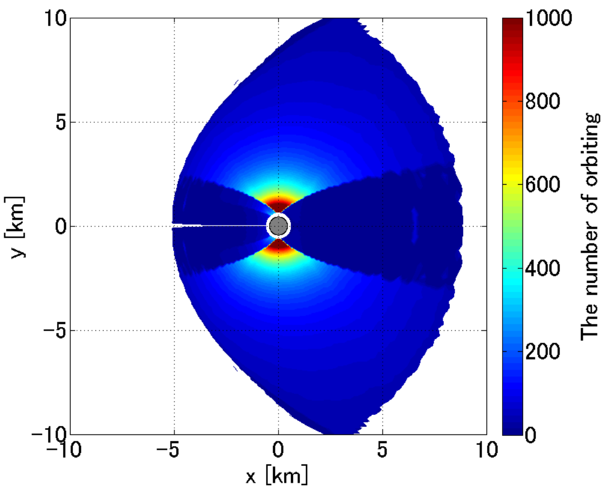


Fig. 4. The number of Orbiting of Orbit around terminator orbit.

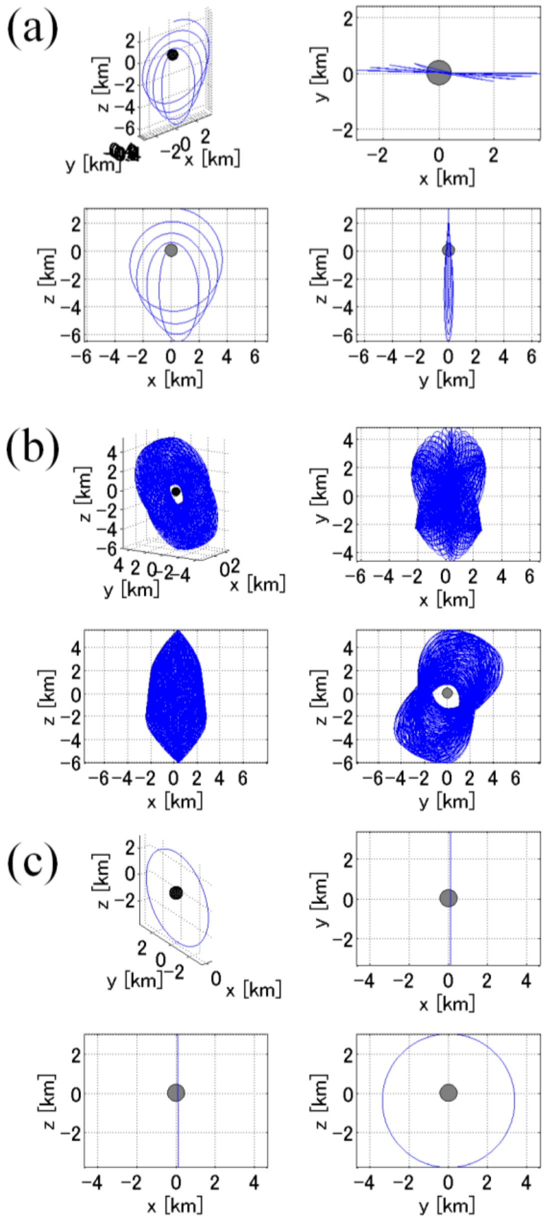


Fig.5. Example of Orbits propagated for 474 days.

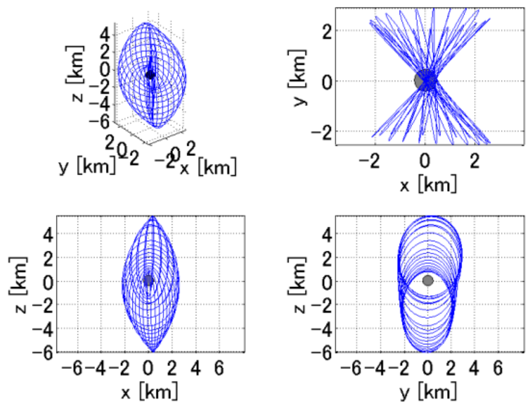


Fig.6. Orbits (b) propagated for 50 days.

3. Analysis of Orbital Element

3.1 The Definition of Orbital Element

This section analyzes the orbital element of QSTO. 6 orbital elements are defined as shown in Fig. 7. It is assumed that the previous section indicated that the initial longitude of the ascending node λ_0 determines if the S/C impacts with Ryugu or not. So, it is interested to know how initial longitude of the ascending node λ_0 affects the behavior of orbit.

3.2. Lagrange Planetary Equations

Solving the Lagrange planetary equations is useful in order to obtain the time history of orbital element. The SRP force

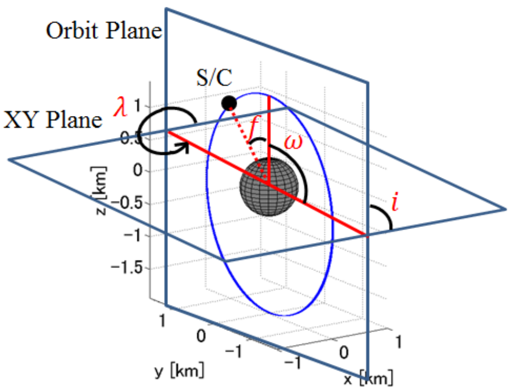


Fig. 7. Definition of orbital elements around an small body.

can be formulated as a disturbing potential and the Lagrange planetary equations can be solved using averaging method.[3] Although these procedures are already investigated in previous study, this paper also shows these process to make this discussion clear. Equations can derive orbital element's differential equations by secular potential caused by perturbation. In this case the perturbation is SRP force and potential function of SRP is expressed by Eq. (5). And then, secular potential can be calculated by averaging this potential function over the S/C mean anomaly as expressed by Eq. (6).

$$R = a_{SRP} r\{\cos(\omega + f)\cos\lambda - \sin(\omega + f)\sin\lambda\cos i\} \quad (5)$$

$$R_s = \frac{1}{2\pi}\int_0^{2\pi} R\, dM \quad (6)$$

Thus, if this secular potential is substituted to the Lagrange planetary equations, these differential equations are derived about orbital elements as shown in Eq.(7)-(12) where e=sinφ, the parameter Λ is indicated by Eq. (13), Λ_C=63.6067 km(kg/m^2)(cm^3/g)$^{1/2}$(AU)$^{1/3}$, and $\tilde{a}$ means semi-major axis normalized by the radius of the small body.

$$\frac{d\lambda}{dv} = -\frac{\Lambda e}{\sqrt{1-e^2}}\sin\omega\sin\lambda - 1 \quad (7)$$

$$\frac{di}{dv} = -\frac{\Lambda e}{\sqrt{1-e^2}}\cos\omega\sin i\sin\lambda \quad (8)$$

$$\frac{de}{dv} = -\Lambda\sqrt{1-e^2}\{\sin\omega\cos\lambda + \cos\omega\cos i\sin\lambda\} \quad (9)$$

$$\frac{d\phi}{dv} = -\Lambda\{\sin\omega\cos\lambda + \cos\omega\cos i\sin\lambda\} \quad (10)$$

$$\frac{d\omega}{dv} = -\frac{\Lambda\sqrt{1-e^2}}{e}\{\cos\omega\cos\lambda - \sin\omega\cos i\sin\lambda\} - \cos i\left(\frac{d\lambda}{dv} + 1\right) \quad (11)$$

$$\frac{da}{dv} = 0 \quad (12)$$

$$\Lambda = \Lambda_C\frac{1+\eta}{r_a B}\sqrt{\frac{\tilde{a}}{P\rho}} \quad (13)$$

If ψ is defined by Eq. (14), ψ characterizes the strength of the SRP force. If ψ is close to 0 deg, SRP force gets small, on the other hand if ψ is close to 90 deg, SRP force gets large. Hayabusa2 and Ryugu properties provide $\psi = 72\sim 89$ [deg] where $\tilde{a} = 1\sim 100$.

$$\psi = \tan^{-1}\Lambda \quad (14)$$

3.3. The Solution of Eccentricity

It is assumed that the boundary which divides the solution space of orbits into impact area and non-impact area is determined by periapsis altitude history of orbit around the small body. Since periapsis altitude is expressed by $a(1-e)$ and semi-major axis a is a constant value as verified by Eq. (12), the history of eccentricity e is most important for obtaining the solution space. In Fig. 8, since $\tilde{a}$ means semi-major axis normalized by the radius of asteroid, this value is equal to 1 means surface of the asteroid. Thus, in the case that periapsis distance history is blue line S/C does not impacts with the small body, however if the minimum of periapsis distance is lower than 1 like the green line, S/C impacts with the asteroid.

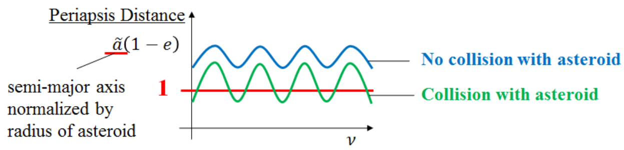


Fig.8. The analogy of periapsis altitude.

These complicated differential equations can be solved only in some special cases using the particular change of variables.[3] When absolute value of the inclination i, argument periapsis ω, and longitude of ascending node λ are $\pi/2$ in condition that indicated by Eq. (14), the eccentricity is frozen as expressed by Eq. (15).

$$i_0 = \frac{\pi}{2},\ \omega_0 = \pm\frac{\pi}{2}, \lambda_0 = \mp\frac{\pi}{2} \quad (14)$$

$$e = \cos\psi \quad (15)$$

The orbit shape is also frozen and this indicates terminator orbit. Note that ψ is a constant value determined by parameters of asteroid and S/C and semi-major axis of orbit.

The QSTO's solution space is characterized by the behavior of the orbit element when the absolute value of inclination i and augment periapsis ω are $\pi/2$ and longitude of ascending node λ is any value expressed by Eq. (16). In this condition, analytical eccentricity solution can be solved and is expressed by Eq. (17).

$$i_0 = \frac{\pi}{2}, \omega_0 = \frac{\pi}{2}, \lambda_0 = \text{any value} \tag{16}$$

$$e = \cos\delta \cos\psi + \sin\delta \sin\psi \cos\left(\frac{\nu - \nu_0}{\cos\psi}\right) \tag{17}$$

where

$$\cos\delta = \sin\psi \cos\phi_0 \sin\lambda_0 + \cos\psi \sin\phi_0$$

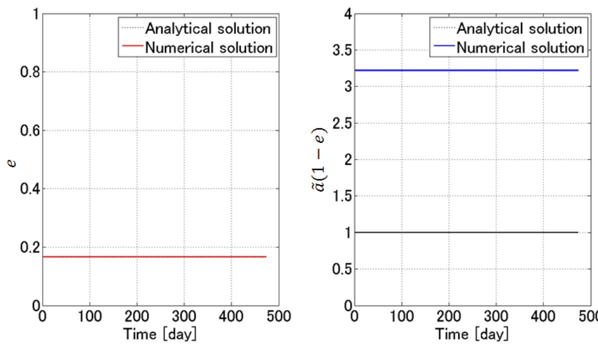
$$\phi_0 = \tan^{-1}\left(\frac{2\cos\psi}{\sin\psi\,(1 - \sin\lambda_0)}\right)$$

δ is also a constant value indicated by ψ. ν indicates asteroid's true anomaly. Therefore, eccentricity history gets simple sine wave and periapsis altitude $a(1-e)$ is also obtained. ν_0 is an integral constant.

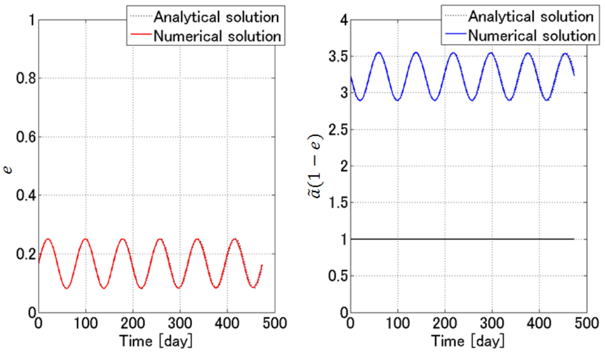
The next subsection shows the result of the comparison of the analytical solution indicated by Eq. (17) and numerical solution in some λ case fixing the size of the orbit.

3.4 Comparison of Analytical and Numerical Solution

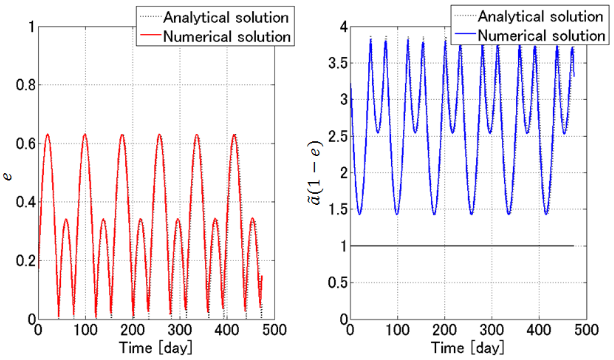
Initial longitude of ascending node λ_0 is set to be 4 cases (90, 85, 60, 30 deg) and the size of orbit z_0 is 1400 m and $\tilde{a}$ is 3.8614. $\lambda_0 = 90$ deg case indicates terminator orbit and only $\lambda_0 = 30$ deg is impact case and other cases are non-impact cases. Orbital elements are approximately calculated every round and numerical solution is derived by them and calculation is continued even if the S/C impacts with the small body. Figure. 9 (a)-(d) shows the history of eccentricity (red line) and periapsis altitude (blue line). Integral constant ν_0 is adjusted to make the initial values of analytical solutions the same values of the numerical solutions. Dash lines indicate the analytical solutions and solid color lines indicate the numerical solutions. In Fig. 9 (a), terminator case, both of eccentricity and periapsis altitude are constant values and analytical and numerical solution completely coincide. On the other hand, it is shown that the behavior of eccentricity and periapsis altitude are sine wave in Fig. 9 (b)-(d). The analytical solutions almost coincide with the numerical solutions. Since solid black line in figures of periapsis altitude indicates the surface of the small body, it is found that only $\lambda_0 = 30$ case indicates the S/C impacts with the small body. On the other hand, periapsis altitudes of other cases can keep over 1 and the S/C can keep its orbit for long duration. Therefore, these results can verify that analytical solution coincides with numerical result.



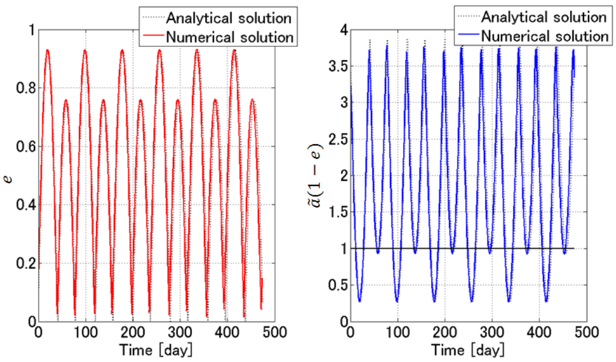
. (a) $\lambda_0 = 90$ deg case



(b) $\lambda_0 = 85$ deg case



(c) $\lambda_0 = 60$ deg case



(d) $\lambda_0 = 30$ deg case

Fig.9. Comparison of numerical and analytical solution of eccentricity and periapsis altitude

Finally, we investigated the comparison of analytical and numerical solution about different size and longitude of ascending node. Figure. 10 shows obtained by global search about the size of orbit and initial longitude of ascending node λ_0 in previous section's analysis. Pink line indicates the boundary which determines if the S/C impacts with the small body or not and is obtained analytically. Analytical solution can be expressed by solving the following equation.

$$F(\psi, \tilde{a}, \lambda_0) = \tilde{a}(1 - e_{\max}) - 1 = 0 \tag{18}$$

$\tilde{a}(1 - e_{\max})$ means the minimum of the periapsis altitude for orbiting around the small body. Thus while F > 0 means that S/C never impacts with the small body and orbit maintain long duration, F<0 means that S/C impacts with the small body and duration is much shorter than the small body's period. This pink line solved analytically coincides with numerical boundary very well. Therefore, this boundary depends on the strength of SRP force ψ, the size of orbit $\tilde{a}$, and initial longitude of ascending node λ_0.

4. Conclusion

First of all, this work examined the solution space about near terminator orbit under strong perturbation environment where SRP is dominant by global search. It is found that there are QSTO that has long duration even if orbit element oscillates. QSTO can provides not only the flexibility of orbit design because it exists much broader region than the terminator orbits but observability of the small body because the S/C can observe the sunlit side of the small body for about a half of its duration. Subsequently, the boundary which determines whether QSTO exists or not is analyzed solving the Lagrange planetary equations. It is verified that the boundary can be obtained by analytical approach. Therefore, if the parameters such as the small body and the S/C, the region of QSTO is useful can be obtained easily.

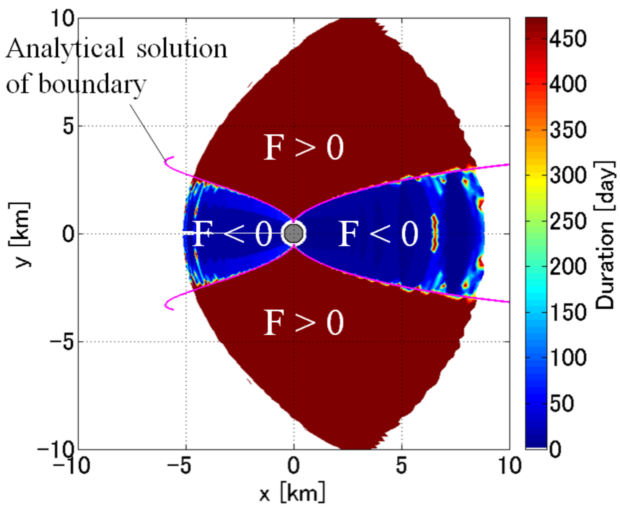


Fig.10. Comparison of boundary that divides impact and non-impact

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